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Solving the mean–variance customer portfolio in Markov chains using iterated quadratic/Lagrange programming: A credit-card customer limits approach

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ABSTRACT

In this paper we present a new mean–variance customer portfolio optimization algorithm for a class of ergodic finite controllable Markov chains. In order to have a realistic result we propose an iterated two-step method for solving the given portfolio constraint problem: (a) the first step is designed to optimize the nonlinear problem using a quadratic programming method for finding the long run fraction of the time that the system is in a given state (segment) and an action (promotion) is chosen and, (b) the second step is designed to find the optimal number of customers using a Lagrange programming approach. Both steps are based on the *c*-variable method to make the problem computationally tractable and obtain the optimal solution for the customer portfolio. The Tikhonov's regularization method is used to ensure the convergence of the objective-function to a single optimal portfolio solution. We prove that the proposed method converges by the Weierstrass theorem: the objective function of the mean–variance customer portfolio problem decreases, it is monotonically non-decreasing and bounded from above. In addition, for solving the customer portfolio problem we consider both, a constant risk-aversion restriction and budget limitations. The constraints imposed by the system produce mixed strategies. Effectiveness of the proposed method is successfully demonstrated theoretically and by a simulated experiment related with credit-card and customer-credit limits approach for a bank.

1. Introduction

1.1. Brief review

Mean–variance analysis for optimal asset allocation is one of the classical results of financial economics (Markowitz, 1952). However, it is well-known by financial managers that firms often perform the portfolio selection in a suboptimal manner. Then, the mix together of huge investment of money and inefficient management of the budget motivates the interest in better understanding of optimal asset allocation. We are focus on optimal asset allocation from the monopoly point of view considering the analysis of customer portfolios selection from a risk management perspective (Ryals, 2003) (we leave the individual customer relationships (Reinartz & Kumar, 2000) point of view of the optimal customer portfolio out of the scope of this paper).

The estimation of the risk is a fundamental topic for (risk-averse) financial managements. Risk management based on diversified portfolio makes it possible to reduce the risk of suffering a large loss and, at the same time, securing a certain level of profitability (Cornuejols & Tutuncu, 2007). Portfolio selection accordingly plays an important role in financial decision making. The risk contribution of each customer to the customer portfolio is taken into account in a customer portfolio valuation applying the Markowitz's theory portfolio selection (Markowitz, 1952). The goal of the customer portfolio is to determine the optimal number of customer types from a value-based risk management perspective. In order to better predict individual customer behavior customers are grouped into segments (Ho, Thomas, Pomroy, & Scherer, 2004). These segments are created by trying to group together customers having similar behavior (Wedel & Kamakura, 2000). Finally, the customer portfolio determines how these segments will be addressed.

In this paper, we consider the single-period budgeted campaign optimization problem (Dar, Mansour, Mirrokni, Muthukrishnan, & Nadav, 2009; Feldman, Muthukrishnan, Pal, & Stein, 2007;

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Muthukrishnan, Pal, & Svitkina, 2007), where the goal is to find the mixed strategies related to the portfolio optimization in order to distribute the budget and to maximize the investment in each applied promotion. The objective function has the form

$$E(X) - \frac{\xi}{2} Var(X) \tag{1}$$

where ξ is a pre-specified risk-aversion parameter. In the Markowitz mean–variance approach the functional (1) is replaced by two different goals: maximize the expected return and to minimize the variance. The intention of this equivalent problem is to focus on the conflicting criteria of return and risk: (a) the return is calculated by computing the expected value of the portfolio return random variable and, (b) the risk is calculated considering the square root of the variance of the portfolio return random variable. It is important to note that the variance generates the same results as the standard deviation, however, they are easier to obtain by computing the means of variance. Each portfolio have a criterion vector describing its risk-return attributes then, the nondominated frontier is the graph with nonnegatively sloped and concave curve containing all portfolio criterion vectors that are nondominated in the standard deviation versus the expected return space. Understanding the standard deviation as risk, the nondominated frontier shows how, if low risk is expected then, low rate of return is obtained, and how, if high rate of return obtained then, high risk is estimated. As a result, a portfolio's criterion vector optimize the utility function if it is on the frontier.

1.2. Related work

Under the assumption of a quadratic utility function the optimal portfolio lies on the mean–variance Pareto front or the Efficient Frontier. A portfolio is optimal if no other possible portfolio is able to improve the optimization criteria. Several methods has been studied in depth in optimization problem theory (Aouni, Colapinto, & La Torre, 2014; Athan & Papalambros, 1996; Bana e Costa & Soares, 2004; Ben Abdelaziz, Aouni, & El-Fayedh, 2007; Best & Hlouskova, 2000; Charnes, Clower, & Kortanek, 1967; Charnes & Cooper, 1977; Das & Dennis, 1998; Deng, Li, & Wang, 2005; Gram & Schyns, 2003; Hirschberger, Qi, & Steuer, 2010; Huang, 2007; Kolm, Ttnc, & Fabozzi, 2014; Korhonen & Yu, 1998; Messac, 1996; Owadally & Landsman, 2013; Qi, Hirschberger, & Steuer, 2009; Stein, Branke, & Schmeck, 2008; Steuer, Qi, & Hirschberger, 2011). Surveys can be found in Marler and Arora (2004), Metaxiotis and Liagkouras (2012) and Rifki and Ono (2012). Moreover, different approaches to tackle portfolio selection have been increasingly applied over the past years (Gutirrez & Magnusson, 2014; Martinsuo, 2013): decision support system (Hu, Wang, Fetch, & Bidanda, 2008; Pendharkar, 2014), fuzzy set theory and multi-criteria (Wei & Chang, 2011; Khalili-Damghani, Sadi-Nezhad, Hosseinzadeh Lotfi, & Tavana, 2013; Khalili-Damghani, 2014; Tavana, Khalili-Damghani, & Abtahi, 2013; Fernandez, Lopez, Mazcorro, Olmedo, & Coello, 2013), R&D portfolio selection (Abbassia, Ashrafi, & Tashnizi, 2014; Bhattacharyya, Kumar, & Kar, 2011). Recent related researches can be found in Aragons-Beltrn, Chaparro-Gonzlez, Pastor-Ferrando, and Pla-Rubio (2014), Dutra, Ribeiro, and Carvalho (2014), Altuntas and Dereli (2015), Liagkouras and Metaxiotis (2015) and Silva, Neves, and Horta (2015).

Most of these algorithms returns either a single solution or set of all efficient portfolios on the Pareto-optimal front, which is a continuous or defined on a sequence of intervals entity (for a discrete parametrization solution see, for example, Qi et al., 2009). Despite the fact that one or only a few solutions may be implemented, it is interesting to look for solutions well-spread on the front. However, to obtain a flexible solution for choosing any point that lies on the Pareto-optimal front, it is needed a continuous parametrization of the front. Therefore, well-spaced points can be shaped on the front to support the process of decision making. A few algorithms attempt to give a continuous representation of the Pareto-optimal front as the result of the optimization process to convex portfolio problems (Best & Hlouskova, 2000; Hirschberger et al., 2010; Korhonen & Yu, 1998; Niedermayer & Niedermayer, 2010; Stein et al., 2008; Steuer et al., 2011). The last four proposed continuous bi-objective and tri-objective problems using parametric quadratic programming.

1.3. Main questions

We noted that many bank mistakes in the most recent years took place for the reason that the managers did not know that portfolio models did not exist or the portfolio models put into operation were ill-behaved. Then, they are forced to play against unnecessary risky positions. The absence of portfolio models or ill-behaved portfolio models may lead managers to make credit-card decision on credit limits that can be catastrophic for the bank. Then, many question arises if a customer portfolio model is put into practice in an institution: (a) does the bank have the required quality information needed to determine the parameters of the customer portfolio? e.g. credit migration (transition matrices), which characterize past changes in credit quality of borrowers play a fundamental role in the Basel Capital Accord (Bank For International Settlements, 2005), where capital requirements are driven in part by ratings migration; (b) does the customer portfolio makes absolute financial sense? i.e. the customer portfolio model uses hypothetical parameters that may lead to incentive distortions for the assets allocation; (c) what are the problems not took into account related to the customer portfolio model? (d) if the behavior of the environment changes, what are the penalties of the bank's credit card decisions according to the customer portfolio? i.e. factors and forces that affect a bank's ability to build and maintain successful relationships with customers: interest rates changes, legal changes, additional regulatory requirements, new competitors behavior, etc.

1.4. Main results

In this work, we present a new algorithm for solving the mean–variance customer portfolio optimization problem using an ergodic finite controllable Markov chains model (Poznyak, Najim, & Gomez-Ramirez, 2000) (for representing the customer behavior and an optimization method for budget allocation). The formulation of the problem is considered as a nonlinear programming problem based on an objective function that is supposed to be (non-obligatory strictly) convex. We introduce the Tikonov's regularization (TR) to ensure the convergence of the objective-function to a single optimal customer portfolio. TR is one of the most popular approaches to solve discrete ill-posed problems represented in the form of a non-obligatory strict convex function. As a result, our method construct the nondominated frontier containing only all contenders for optimality. In order to have a realistic result we propose an iterated quadratic/Lagrange programming approach to obtain the optimal solution for the mean–variance customer portfolio. The proposed method is an iterative two-step procedure for solving the given quadratic problem (1): the first step is designed to optimize the nonlinear problem using a quadratic programming method for finding the long run fraction of the time that the system is in a given state (segment) and an action (promotion) is chosen (represented by the variable c) and, the second step is designed for finding the optimal number of customers using a Lagrange programming approach. At each iteration of the procedure the objective function of the mean–variance customer portfolio problem (1)