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Optimal Diamond–Dybvig mechanism in large economies with aggregate uncertainty

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ABSTRACT

This paper characterizes the direct mechanism which implements the constrained optimal outcome in a version of Diamond and Dybvig (1983) with aggregate uncertainty and a continuum of agents. Using this result, numerical examples where the best direct mechanism has a bank-run-equilibrium are easily obtained.

1. Introduction

At least since Diamond and Dybvig (1983) (hereafter DD), bank-runs have been thought of as arising from multiple equilibria. However, it has not been easy to show that the direct mechanism which has the best no-run-equilibrium also has a run-equilibrium in settings where the bank can commit to a mechanism.[1] In environments without aggregate uncertainty, in which the number of impatient agents is known, a suspension scheme strongly implements the best outcome. In environments with aggregate uncertainty, the result depends on the details of the model. Green and Lin (2003) (hereafter GL) provide a negative answer in an environment with a finite number of agents and *i.i.d.* preference shocks when agents are informed about their position in the queue before deciding whether to withdraw or not. Peck and Shell (2003) and Ennis and Keister (2009b) (hereafter PS and EK) provide examples with a finite (and small) number of agents where the best direct mechanism has a run-equilibrium. While agents are uninformed about their position in the queue in PS, EK consider both cases. EK also show by examples that the result obtained by GL does not hold if the preference shocks are correlated. In a setting similar to PS, Cavalcanti et al. (2011) (hereafter CBM) show that a run-equilibrium exists under the best mechanism if the population is large enough. However, they restrict their analysis to an environment with *i.i.d.* preference shocks. The law of

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[1] Here and throughout the paper I consider only environments with commitment. For a discussion on bank-runs in a setting without commitment see Ennis and Keister (2009a).

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large numbers implies that this economy is well approximated by an economy without aggregate uncertainty when the population is large. Therefore, CBM is not suitable for the analysis of aggregate uncertainty.

My model has aggregate risk, a nonatomic measure of agents, and the information structure in PS. The uncertainty is modelled as follows. The impatient fraction of the population is a random variable. Conditional on its realization, the event that a person is patient or impatient is *i.i.d.* across the population. In this setting the preference shocks are unconditionally correlated across people. The virtue of this specification is that results from variational calculus can be used to characterize the direct mechanism which has the best no-run-equilibrium. In addition, it is easy to check whether the mechanism has a run-equilibrium.

Previous papers have shown similar features in economies with a finite number of agents. However, in those papers the number of equations and unknowns is proportional to the number of agents, which is why PS only provide examples with few people.[2] EK show that dynamic programming can be used to compute the constrained optimal allocation in large economies if the constrained optimal allocation coincides with the unconstrained one. However, for some economies, the unconstrained optimal allocation is not the constrained optimal one. In this case, the EK dynamic programming approach cannot be used while the method I propose here is still valid.

2. The model

In this section I describe the environment, the class of mechanisms, and the kind of equilibrium that may arise given a mechanism.

2.1. Environment

There are three periods, 0, 1, 2. In period 0, agents have y units of wealth which they invest to consume in periods 1 and 2. The investment technology is the same as DD, an amount x invested in period 0, pays gross return 1 if liquidated in period 1 and gross return $R>1$ if liquidated in period 2. There is a continuum of people with nonatomic unit measure and they can be either *patient* or *impatient*. The utility of an individual of type *impatient* is $u(c_1)$, while that of type *patient* is $v(c_1+c_2)$, where (c_1, c_2) is consumption in periods 1 and 2 respectively. The functions $u, v : \mathbb{R}_+ \to \mathbb{R}$ are strictly increasing, strictly concave and twice continuously differentiable. Let $\alpha \in [0,1]$ denote the fraction of agents who are *impatient*, where α is a random variable with cumulative distribution function F and a continuous density function f which satisfies $f(x)>0$ for all $x \in (0,1)$. Conditional on the realization of α, the event that an individual is *impatient* follows a Bernoulli distribution with parameter α.

I study what PS call the post-deposit game. That is, at the outset (*i.e.* at the beginning of period 0), I assume that all individuals deposit their resources in the bank. Then I focus solely on the agent's decision to withdraw or not once they know the realization of their types. In period 0, individuals face uncertainty about the value of α and their types. They only know the distribution of α, which is common knowledge. In the beginning of period 1, each individual observes his own type (which is private information). No one observes the realization of α. Then, agents simultaneously decide whether to withdraw or not. The bank serves the withdrawal requests of the individuals in a random sequence, which the literature refers to as the sequential service constraint. An individual's position in the queue is uniformly distributed among people who decide to withdraw. Although they arrive at the bank in sequence, the rate of arrivals cannot be measured by the bank.[3] After all withdrawal payments are made, what is left in the bank pays a gross return of R. In period 2 the bank distributes the amount left to those who did not withdraw in period 1. Fig. 1 depicts the sequence of actions.

An important difference between this paper and part of the literature is the assumption that only people who decides to withdraw report to the bank. The purpose of this assumption is twofold. First, it seems realistic. We do not observe depositors without liquidity needs contacting the bank to announce that they are not withdrawing. The second reason is technical. If all agents contact the bank, it is possible to design a mechanism that uses an arbitrarily small fraction of the population to exactly estimate the realization of the aggregate uncertainty. I rule out this possibility in order to study the effect of aggregate uncertainty on *bank runs* in large economies.

2.2. Direct mechanisms

I focus on direct mechanisms. The revelation principle implies that this is without loss of generality with respect to finding the best weakly implementable outcome. Because withdrawing in period 1 is equivalent to claiming to be of type *impatient*, I will not distinguish between those two actions. I also do not distinguish between withdrawing in period 2 or revealing to be of type *patient*. Because the bank does not observe α, the mechanism describes only how much an individual consumes at date 1 as a function of the position $z \in [0,1]$ in the queue.

[2] PS have an example with 300 agents, but they restrict the number of impatient types to be either 100, 200 or 300. This restriction implies that this economy is equivalent to one with only 3 agents.

[3] This assumption can be rationalized by assuming that the time interval in which agents arrive varies proportionally to the number of agents visiting the bank.