NORTH-HOLLAND

International Review of Financial Analysis
11 (2002) 159–181

IRFA
INTERNATIONAL REVIEW OF
Financial Analysis

The stochastic-volatility American put option of banks' credit line commitments: Valuation and policy implications

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Abstract

This paper investigates commitment credit risk and valuation in connection with their risk-adjusted balance used in computing the bank's capital requirement mandated by the Bank for International Settlements (BIS). In a two-factor model of the marked-to-market value of the credit line (CL), x, and its mean-reverting volatility, V, the value of the American commitment put is obtained as the sum of a Fourier-based solution for the European put and a quadratic approximation for the early-exercise premium. Once computed, the put value is combined with the line fees and a conditional exercise-cum-takedown proportion to determine the commitment net value and the bank's exposure to commitment credit risk. A comparison between the stochastic and constant volatility option models reveals that correlation rather than stochastic volatility forms the greater source of bias: the impact of the correlation-generated skewness on the distribution of the CL marked-to-market value is more significant than that of the σ-generated kurtosis. The random-volatility model is next used to ascertain how commitment credit risk affects banks' capital requirement. According to the BIS accounting-based procedure, the risk-adjusted balance of short-term commitments is nil; this is not the case when the same risk-adjusted balance is computed by way of the option-based procedure. Beyond capital sufficiency, the approach also determines the impact of commitment credit risk on the bank's future profits.

Keywords: Stochastic-volatility American commitment put option; Commitment net value and banks' exposure to commitment credit risk

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PII: S1057-5219(02)00073-X

1. Introduction

Banks carry off-balance-sheet substantial amounts of short- and long-term credit line (CL) commitments, which, due to credit risk, are subject to the capital adequacy guideline mandated by the Bank for International Settlements (BIS). This situation raises two questions: (1) What is the value of the put option embedded in credit commitments? (2) Do banks incur any liabilities when offering this type of credit, and if so, how is their exposure to commitment risk computed?

Thakor, Hong, and Greenbaum (1981) have shown the isomorphic correspondence between commitment contracts and equity put options: When the rate defined in a credit line commitment is lower than that on an equivalent spot loan, the borrower receives the line face value but is only indebted for its lower marked-to-market value (from now on, the latter will be referred to as the "indebtedness value"). The borrower's claim on the lending bank constitutes an embedded, yet valuable, commitment put option. The aggregate face value of unused commitments is reported as an off-balance-sheet entry to the bank's annual consolidated balance sheet and is subject to regular (monthly, quarterly, and, by law, annual) audits. Yet, at the annual reporting date, the time remaining to commitment expiry is less than the initial term for many of the outstanding commitments; borrowers, moreover, have the right to draw on the CLs at any point in time, and even in part(s). To account for the commitment early exercise with partial line funding, we propose to model CL commitments as American (instead of European) put options. Their value thus captures the bank's notional liability for carrying off-balance-sheet commitments at the annual audit date. In this research, we examine the most prevalent type of CL commitments, those with a floating-rate formula devised as "stochastic index cost of funds plus a fixed forward markup." Among those, we concentrate on the class of "prime-rate" commitments with an original term to maturity less than 1 year. These short-term commitments finance working capital, trade, and commerce.

In recent years, several researchers have derived alternative formulas for valuing bank CL commitments. Bartter and Rendleman (1979), Ho and Saunders (1983), and Thakor et al. (1981) derived option-like expressions for fixed-rate commitments; Chateau (1990, 1995) and Thakor (1982) obtained valuation formulas for variable-rate commitments and Hawkins (1982) priced revolving CLs. All chose, however, to retain the assumption that the volatility of the indebtedness-value diffusion is constant. In actuality, the indebtedness-value volatility may vary stochastically and may or may not be correlated with the indebtedness value itself. Fortunately, there have been advances in this area of research. Numerical solutions for stochastic-volatility (SV) stock returns had been proposed, among others, by Dumas, Fleming, and Whaley (1998), Scott (1987), and Wiggins (1987). Dothan (1987) and Hull and White (1987, 1988) also provided power series approximations that were mathematically tractable and easy to implement. Next, Bakshi, Cao, and Chen (1997), Bates (1996), Heston (1993), Scott (1997), and Stein and Stein (1991) offered exact solutions based on the Fourier inversion method. More recently, Bates (2000) and Das and Sundaram (1999) examined the components of the stochastic anomaly. In the latter case, the idea was to determine whether the return distributions exhibited both skewness and excess kurtosis. The balance of the empirical evidence reported so far seems to imply that: (1) SV models do explain volatility