Insurance: Mathematics and Economics 53 (2013) 551–568

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Insurance demand and welfare-maximizing risk capital—Some hints for the regulator in the case of exponential preferences and exponential claims

Daniel Burren

ProMaSta, www.promasta.com, Singapore

HIGHLIGHTS

- We propose a tractable model for insurance demand in continuous time.
- For exponential claims, we obtain closed-form solutions.
- Welfare-maximizing risk capital is discussed considering capital costs.
- A risk-neutral agent might buy insurance even if the premium is above the expected loss.
- We provide a new insight on the Bernoulli principle within a static model.

ARTICLE INFO

Article history:
Received September 2012
Received in revised form
March 2013
Accepted 3 August 2013

JEL classification:
C61
D60
D81

Keywords:
Welfare-maximizing risk capital
Insurance demand
Risk theory
Optimal stochastic control

ABSTRACT

We propose two models to analyze welfare-maximizing capital requirements for insurance companies considering that capital is costly and therefore affecting the premium. Within a continuous-time model, we derive insurance demand and welfare as a function of personal wealth, the insurance company's wealth, and the claims process, and compare them to their counterparts in a static model. Besides discussing welfare-maximizing capital, we provide some new insights on insurance demand.

1. Introduction

Across Europe, new regulatory frameworks require insurance companies to back their liabilities based on a one-year distribution of assets and liabilities. In Switzerland, the amount of risk capital the insurance company must hold is given by the 99th tail value at risk (TVaR) of this distribution. This means that the insurance company has to have enough capital to pay for the worst year in 100 years time. The legal framework is called the Swiss Solvency Test (SST). Since January 2011, insurance companies must submit the results of their SST to the financial authority FINMA. A similar framework, called Solvency II, exists within the European Union. It was expected to be fully implemented in 2013 but might now be pushed back to 2016. The capital requirement according to Solvency II is given by the 99.5th value at risk (VaR), which means that an insurance company goes bankrupt at most once in a 200-year period. Other parts of the world are expected to follow. The Bermuda Monetary Authority, for example, is preparing to achieve regulatory equivalence with Europe's Solvency II. In Singapore, the Monetary Authority is embarking on a review of its existing risk-based capital framework.

There have been vibrant discussions about risk measures (Acerbi and Tasche, 2002; Artzner et al., 1999; Delbaen, 2000; Föllmer and Schied, 2002; Hamel and Heyde, 2010; Jaschke, 2001; Jouini et al., 2004), about asset–liability-management and optimal risk capital from an investor's point of view (Buch and Dorfleitner, 2008; Filipovic and Kupper, 2008; Schmautz and Lampenius, 2011; Stein, 2010), and about capital allocations to sub-portfolios (Buch and Dorfleitner, 2008; Dhaene et al., 2012; Tsanakas, 2009). Powers (1995) looks at optimal regulation considering insolvency costs. Dhaene et al. (2008) find that the VaR is the optimal risk measure if

E-mail address: danielburren@gmail.com.

0167-6687/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.insmatheco.2013.08.001

the objective of the regulator is to minimize a cost function taking account of the risk and of capital costs. However, the question of how to determine regulatory capital requirements in order to maximize welfare has, to our knowledge, not yet been treated. If capital requirements are too weak, an agent might no longer buy insurance because of the insolvency risk. If, however, capital requirements are too strict, then insurance might simply become too expensive.

We propose two models for doing welfare analysis, a static one and a dynamic one in continuous time with an optimizing agent. Within both models we study how insurance demand depends on the solvency of the insurance company considering that capital costs affect the premium. While the static model is more representable for insurance demand of a private household, the dynamic model allows us to take into account the default risk of the insurance buyer, and is therefore more suitable for a business buying insurance (for example an insurance company seeking reinsurance or a business insuring against business interruption or liability claims). For both models, we consider a standard risk process where both the frequency and severity of claims are modeled, and study the impact of these two components on demand. We present examples for which welfare and demand in the dynamic model can be obtained in closed form by solving either differential equations or a hyperbolic partial differential equation. We further relate these welfare and demand functions to their counterparts in the static model. Along the way we present a new insight on the Bernoulli principle, and provide circumstances where it can be optimal for a risk-neutral agent to buy full coverage even if the premium is above the expected loss.

Based on different calibrations of our models, we find the following. First, an insurance buyer who faces a high personal default risk due to small wealth is willing to pay a higher premium to insure against high-frequency low-severity claims than to insure against low-frequency high-severity claims. With increasing wealth, the preferences change. A rich agent is more prone to buy insurance against low-frequency high-severity claims and would insure against high-frequency low-severity claims only if the premium is low. Second, we find that the welfare-maximizing capital is increasing in all of the following components: the size of insurance companies, risk aversion, and insurance buyer's default risk due to claims. Finally, we can easily find a calibration such that capital requirements at a confidence level of 99.5% (as in Solvency II) are optimal.

We next provide a short literature review. Then, Section 2 presents the static model and derives the welfare and demand functions. Section 3 presents the dynamic model. Section 4 presents numerical implementations, and Section 5 concludes.

1.1. Literature review

Our paper contributes to existing literature on insurance and risk sharing. Pioneering work consists of Mossin (1968), Arrow (1974), Borch (1975), who derived optimal insurance arrangements in one-period models, and further Borch (1962), Bühlmann (1980), Jouini et al. (2008) who characterized equilibriums in risk-sharing markets. This research, however, abstracts from counterparty risk and capital considerations.

Capital considerations were introduced by Winter (1988), Gron (1994), Cagle and Harrington (1995) as an explanation for the underwriting cycle. Cummins and Danzon (1997) added default risk as a factor in insurance demand. They found that a drop in insurers' capital can have two effects on the premium: an increasing one due to capacity constraints (supply-side effect), and a decreasing one due to reduced solvency (demand-side effect). Gründl et al. (2006), Yow and Sherris (2008), Schlütter and Gründl (2011) also included default risk in insurance demand and derived implications for capital allocation schemes which maximize the welfare of shareholders. They assumed an exogenous demand function, as this was sufficient for their research questions. Our approach is different. We derive demand by utility maximization because the center of our study is insurance buyers' welfare.

Obviously, rational insurance demand for non-performing contracts has been derived before; see for example Schlesinger and Schulenburg (1987), Doherty and Schlesinger (1990), Mahu and Wright (2004), Biffis and Millossovich (2012). This literature, however, focuses on demand-related questions like designing optimal contracts or the influence of risk aversion on demand in the presence of counterparty risk. It does not derive any policy implications for the regulator. Moreover, in that literature the premium is often assumed to be actuarially fair and abstracts from capital costs. Closer to our work is Charpentier and Le Maux (2010), who find that the government should encourage the emergence of a monopoly in the natural catastrophe insurance market. But that paper does not provide an answer to the question of welfare-maximizing risk capital.

All of the above papers employ a one-period framework. Yet, our paper is not the first to study insurance demand in a dynamic model using stochastic control techniques. Bruhn and Steffensen (2011), for example, solved for optimal life insurance purchase contingent on the age and wealth of the individual. Further, Moore and Wright (2006) presented optimal consumption, investment, and insurance strategies for an individual who seeks to maximize his/her expected utility of consumption. As we do in our paper, they assume that the claims follow a compound Poisson process, and they present an example for exponential utility. Touzi (2000) studied the problem of maximizing expected utility from terminal wealth when the agent can buy insurance against shocks from a marked point process. Cao and Zeng (2012) derived the optimal reinsurance and investment for an insurer that minimizes its ruin probability. While some of these papers considered more general insurance policies than the one in our paper, they did not model the default risk of the (re)insurance provider, which means that an important component for discussing welfare-maximizing risk capital is missing. Insurer default is included in the model of Liu and Myers (2012). They analyzed the demand for agricultural insurance by risk-averse farmers who can borrow and lend subject to a liquidity constraint and who face the risk of insurer default. However, they assume a constant ruin probability and do not consider that the solvency risk depends on capital.

Complementary to the cited research and our paper is the experimental work of Zimmer et al. (2009, 2011) and the references therein, which shows that people dislike insurance contracts with default risk and that insurance demand is very sensitive to the insurer's solvency. And we mention Grace and Phillips (2007), who investigated efficient insurance regulation.

2. Static model

In order to assess the implications of regulatory risk capital on the insurance market, we consider an economic model with three players: insurance buyer, insurance supplier, and the financial authority.

In a first step, we consider a one-period setting where the insurance buyer maximizes the expected utility of final wealth. More precisely, we suppose that the final wealth of the optimizing agent is

$$\mu - \pi(u) - \sum_{i=1}^{N} X_i + I(u), \tag{1}$$

where μ is the agent's income net of all expenditures except insurance, π the premium payment, I the indemnity, u a parameter chosen by the agent, N the number of claims, and X the claim intensity. We write the problem of the optimizing insurance buyer as