Comments on the EOQ model under retailer partial trade credit policy in the supply chain

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Huang and Hsu [2007. An EOQ model under retailer partial trade credit policy in supply chain. International Journal of Production Economics, doi:10.1016/j.ijpe.2007.05.014] investigate the inventory system as a cost minimization problem to determine the retailer's optimal inventory policy under the supply chain. They develop two easy-to-use theorems to locate the optimal inventory policy for the retailer. Although their inventory models are correct and interesting, processes of arguments to derive those theorems to find the optimal inventory policy are not complete. So the main purpose of this paper is to overcome these shortcomings and present complete proofs for Huang and Hsu (2007).

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1. Introduction

Goyal (1985) is the first person to develop a single-item inventory model under permissible delay in payments. He assumes that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to the customer. Huang (2003) modifies Goyal (1985) to assume that the retailer will also adopt the trade credit policy to stimulate customer's demands. The viewpoint of Huang (2003) is called two levels of trade credit. Huang (2006) incorporates Huang (2003) and Teng (2002) to investigate two levels of trade credit and limited storage space. Recently, Liao (2007) considers an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. Chung and Huang (2007) explore the optimal retailer's ordering policies for deteriorating items with limited storage capacity under trade credit financing. Subsequently, Huang (2007) combines both Huang (2003) and Chung and Huang (2003) to develop an optimal retailer's replenishment decision in the EPQ model under two levels of trade credit policy. Furthermore, Huang and Hsu (2007) consider another kind of two levels of trade credit and present an EOQ model under the retailer partial trade credit policy in supply chain. Basically, their inventory model is correct and interesting. They establish two easy-to-use theorems to characterize the optimal policy; however, processes of arguments to derive those theorems are not complete. So, the main purpose of this paper is to overcome those shortcomings and present complete proofs for Huang and Hsu (2007).

2. Model formulation

The following notation and assumptions will be used throughout the paper.

Notation:

\[ D = \text{demand rate per year} \]
\[ A = \text{ordering cost per order} \]
\[ c = \text{unit purchasing price} \]
\[ s = \text{unit selling price, } s > c \]
\[ h = \text{unit stock holding cost per year excluding interest charges} \]
\[ z = \text{the customer’s fraction of the total amount owed payable at the time of placing an order offered by retailer, } 0 \leq z \leq 1 \]

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\( l_e \) = interest earned per $ per year  
\( l_k \) = interest charged per $ in stocks per year by the supplier  
\( M \) = the retailer’s trade credit period offered by the supplier  
\( N \) = the customer’s trade credit period offered by the retailer in years  
\( T \) = the cycle time in years  
\( TRC(T) \) = the annual total relevant cost, which is a function of \( T \)  
\( T^* \) = the optimal cycle time of \( TRC(T) \)  
\( Q^* \) = the optimal order quantity = \( DT^* \)  

Assumptions:  
(1) Demand rate, \( D \), is known and constant.  
(2) Shortages are not allowed.  
(3) Time horizon is infinite.  
(4) Replenishments are instantaneous.  
(5) The supplier offers the full trade credit to the retailer.  
(6) The retailer just offers the partial trade credit to his/her customer. Hence, his/her customer must make a partial payment to the retailer when the items are sold. Then his/her customer must pay off the remaining balance at the end of the trade credit period offered by the retailer. That is, the retailer can accumulate interest from his/her customer payment with rate \( l_e \).  

The annual total relevant cost consists of the following elements. Two situations may arise. (I) \( M \geq N \) and (II) \( M < N \).  

Case I: Suppose that \( M \geq N \). In this case, Huang and Hsu (2007) show that the annual total relevant cost \( TRC(T) \) can be expressed as follows:  
\[
TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } T \geq M \\ 
TRC_2(T) & \text{if } N < T < M \\ 
TRC_3(T) & \text{if } 0 < T < N 
\end{cases} 
\]  
(1a–c)  

where  
\[
TRC_1(T) = \frac{A}{T} + \frac{DTh}{2} + cl_kD(T - M)^2 / 2T - sl_kD[M^2 - (1 - z)N^2] / 2T 
\]  
(2)  
\[
TRC_2(T) = \frac{A}{T} + \frac{DTh}{2} - sl_kD[2MT - (1 - z)N^2 - T^2] / 2T 
\]  
(3)  
and  
\[
TRC_3(T) = \frac{A}{T} + \frac{DTh}{2} - sl_kD \left[ M - (1 - z)N - \frac{zT}{2} \right] 
\]  
(4)  

Since \( TRC_1(M) = TRC_2(M) \) and \( TRC_2(N) = TRC_3(N) \), \( TRC(T) \) is continuous and well-defined. All \( TRC_1(T), TRC_2(T), TRC_3(T) \) and \( TRC(T) \) are defined on \( T > 0 \).  

Case II: Suppose that \( M < N \). In this case, Huang and Hsu (2007) show that the annual total relevant cost \( TRC(T) \) can be expressed as follows:  
\[
TRC(T) = \begin{cases} 
TRC_4(T) & \text{if } T \geq M \\ 
TRC_5(T) & \text{if } 0 < T < M 
\end{cases} 
\]  
(5a,b)  

where  
\[
TRC_4(T) = \frac{A}{T} + \frac{DTh}{2} + cl_kD(T - M)^2 / 2T - sl_kD(M^2 - (1 - z)N^2) / 2T 
\]  
(6)  
and  
\[
TRC_5(T) = \frac{A}{T} + \frac{DTh}{2} - sl_kD \left[ zM - \frac{zT}{2} \right] 
\]  
(7)  

Since \( TRC_4(M) = TRC_5(M) \), \( TRC(T) \) is continuous and well-defined. All \( TRC_4(T), TRC_5(T) \) and \( TRC(T) \) are defined on \( T > 0 \).  

3. The convexity of \( TRC(T) \)  

Case I: \( M \geq N \). Eqs. (2)–(4) yield  
\[
TRC_1(T) = - \left[ \frac{2A + cDM^2l_k - sDI_e(M^2 - (1 - z)N^2)}{2T^2} \right] + D \left( \frac{h + cl_k}{2} \right) 
\]  
(8)  
\[
TRC_2(T) = \frac{2A + cDM^2l_k - sDI_e(M^2 - (1 - z)N^2)}{T^2} 
\]  
(9)  
\[
TRC_3(T) = - \frac{2A + sD(1 - z)N^2l_e}{2T^2} + D \left( \frac{h + sl_e}{2} \right) 
\]  
(10)  
\[
TRC_3(T) = \frac{2A + sD(1 - z)N^2l_e}{T^2} > 0 
\]  
(11)  
\[
TRC_3(T) = \frac{-A}{T^2} + D \left( \frac{h + sl_e}{2} \right) 
\]  
(12)  
and  
\[
TRC_3(T) = \frac{2A}{T^2} > 0 
\]  
(13)  

Eqs. (11) and (13) imply that \( TRC_2(T) \) and \( TRC_3(T) \) are convex on \( T > 0 \) and Eq. (6) implies that \( TRC_1(T) \) is convex on \( T > 0 \) when \( 2A + cDM^2l_k - sDI_e(M^2 - (1 - z)N^2) > 0 \). Furthermore, we have \( TRC_1(M) = TRC_2(M) \) and \( TRC_2(N) = TRC_3(N) \). Therefore, Eqs. (1a–c) imply that \( TRC(T) \) is convex on \( T > 0 \) when \( 2A + cDM^2l_k - sDI_e(M^2 - (1 - z)N^2) > 0 \).  

Let  
\[
G = 2A + cDM^2l_k - sDI_e(M^2 - (1 - z)N^2) 
\]  
(14)  

Then, we have the following results.  

Lemma 1. Suppose that \( M \geq N \). Hence,  

(A) If \( G \leq 0 \), then \( TRC(T) \) is convex on \( [0, M] \) and concave on \( [M, \infty) \). Furthermore, if \( G < 0 \), then \( TRC_1(T) > 0 \) and \( TRC_1(T) \) is increasing on \( T > 0 \).  

(B) If \( G > 0 \), then \( TRC(T) \) is convex on \( (0, \infty) \).
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