Applied Mathematical Modelling 37 (2013) 3322–3339

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Retailer's optimal replenishment and payment policies in the EPQ model under cash discount and two-level trade credit policy

Hairong Feng [a,*], Jun Li [b], Da Zhao [b]

[a] Business School, Sichuan Normal University, Chengdu, Sichuan 610101, PR China
[b] School of Economics and Management, Southwest Jiaotong University, Chengdu, Sichuan 610031, PR China

ARTICLE INFO

Article history:
Received 26 July 2010
Received in revised form 28 May 2012
Accepted 3 July 2012
Available online 17 August 2012

Keywords:
Inventory
EPQ
Two levels of trade credit
Cash discount

ABSTRACT

The main purpose of this paper is to investigate the retailer's optimal cycle time and optimal payment time under the supplier's cash discount and trade credit policy within the economic production quantity (EPQ) framework. In this paper, we assume that the retailer will provide a full trade credit to his/her good credit customers and request his/her bad credit customers pay for the items as soon as receiving them. Under this assumption, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal payment time under the replenishment rate is finite. Then, an algorithm is established to obtain the optimal strategy. Finally, numerical examples are given to illustrate the theoretical results and obtain some managerial phenomena.

1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer must pay for the purchased items as soon as items are received. This is not always true in the actual business world. In fact, the supplier usually permits the retailer a delay of a fixed time period to settle the total amount owed to him. During such period, the retailer can sell the goods, accumulate revenue and earn interest. Over years, a number of researches have been published which dealt with the inventory model under trade credit. Goyal [1] suggested a mathematical model for obtaining the economic order quantity under permissible delay in payments. Aggarwal and Jaggi [2] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal et al. [3] extended this issue with allowable shortage. Chung and Huang [4] extended Goyal's model by considering the units are replenished at a finite rate. Teng [5] amended Goyal's model by considering the difference between unit price and unit cost, and found that the economic replenishment interval and order quantity decrease under the permissible delay in payments in certain cases. Chung and Huang [6] extended Goyal's model by considering allowable shortage and presented a theorem to determine the optimal order quantity.

However, in most business transactions, the supplier not only allows a certain fixed period for settling the account but may also offer a cash discount to encourage the retailer to pay for his purchases quickly. The retailer can obtain the cash discount when the payment is paid within cash discount period offered by the supplier. Otherwise, the retailer will pay full payment within the trade credit period. In general, the cash discount period is shorter than the trade credit period. For example, the supplier agrees to a 2% discount off the retailer's purchasing price if payment is made within 10 days. Otherwise, full payment is required within 30 days after the delivery, and the credit term in financial management is usually denoted as "2/10, net 30" (Brigham [7, p. 741]). Huang and Chung [8] extended Goyal's [1] model with cash discount and determined

* Corresponding author.
E-mail address: hairongfeng@hotmail.com (H. Feng).

0307-904X/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2012.07.012

the optimal cycle time and the optimal payment policy in the EOQ model under cash discount and trade credit so that the annual total relevant cost is minimized. Huang [9] extended Huang and Chung's [8] model by considering the difference between unit price and unit cost. Huang [10] extended Huang and Chung's [8] model by considering the replenishment rate is finite. Ouyang et al. [11] established an EOQ model with limited storage capacity, in which the supplier provides cash discount and permissible delay in payments for the retailer. Ho et al. [12] formulated an integrated supplier–buyer inventory model with the assumptions that the market demand is sensitive to the retail price and the supplier offers two payment options: trade credit and early-payments with discount price to the buyer.

All of the above models assumed that the supplier would offer the retailer a permissible delay of payments. That is one level of trade credit. Huang [13] pointed out that the retailer may also adopt the trade credit policy to stimulate his/her customer demand in most business transactions. Huang [14] defined this situation as two levels of trade credit policy, and incorporated both Chung and Huang [4] and Huang [13] to investigate the optimal retailer's replenishment decisions with two levels of trade credit policy in the EPQ framework. Teng and Chang [15] overcame a shortcoming in Huang's [14] model and proposed the generalized formulation to the problem. Huang and Hsu [16] extended Huang's [13] model by considering the retailer just offers the partial trade credit to his/her customer. Teng [17] established an inventory lot-sizing model for a retailer who receives a full trade credit from its supplier, and offers either a partial trade credit to its bad credit customers or a full trade credit to its good credit customers.

Therefore, in this study, we try to adopt the payment rule discussed in Teng [17] and the payment rule discussed in Huang and Chung [8] to develop an inventory model under the replenishment rate is finite. Then, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal payment time under cash discount and two-level trade credit within the EPQ framework. Furthermore, an algorithm is developed to determinate the optimal replenishment policy for the retailer. Finally, numerical examples are given to illustrate the theoretical results. In addition, we obtain a lot of managerial insights from numerical examples.

2. Notation and assumption

The following notation is used throughout this paper.

D	annual demand rate
P	annual replenishment rate, $P > D$
Q	order size
c	purchasing cost per unit
K	fixed ordering cost per order
s	selling price per unit
h	stock-holding cost per unit per year
I_d	interest earned per \$ per year
I_c	interest charged per \$ per year
M_1	retailer's fixed period of cash discount
M_2	retailer's fixed period of permissible delay in settling accounts, with $M_2 > M_1$
N	customer's fixed period of permissible delay in settling accounts
δ	cash discount rate, $0 < \delta < 1$
α	the fraction of the customers who pay for the items immediately upon receiving them, $0 \leqslant \alpha \leqslant 1$
$1-\alpha$	the portion of the customers who receive a permissible delay of payment.
T	inventory cycle length of each cycle
T^*	optimal replenishment cycle time
Q^*	optimal order quantity

In addition, the following assumptions are made in the model:

(1) There is single-supplier and single-retailer for a single product in this model.
(2) The demand rate is known, constant, and continuous, and the replenishment rate is known and constant.
(3) The lead-time is zero.
(4) Shortages are not allowed.
(5) To speed up cash in flow and reduce the risk of cash flow shortage, the supplier offers a discount, δ $(0 < \delta < 1)$, off the retailer's unit purchasing price, if the retailer settles the account at time M_1. Otherwise, the full price of the purchase is charged.
(6) During the credit period (i.e., M_1 or M_2), the retailer sells the items and uses the sales revenue to earn interest at a rate of I_d. At the end of this period, the retailer pays off all purchasing cost to the supplier and starts paying for the interest charges for the items in stock with rate I_c.
(7) $s \geqslant c$, $I_c \geqslant I_d$, and $c(1-\delta)I_c \geqslant sI_d$.