



# Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy

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## ABSTRACT

The traditional inventory of the economic order quantity model assumes perfect items in an ordered lot and an infinite replenishment rate. However, such conditions are rare in actual production environments. Additionally, most studies of this problem have only considered suppliers offering the wholesaler a grace period. In practice, wholesalers often extend a fixed credit period to downstream customers as well. This study therefore proposes a production model for a lot-size inventory system with finite production rate and defective quality under the condition of two-level trade credit policy and the condition that defective items involve both imperfect quality and scrap items. Thus, optimal wholesaler replenishment decisions can be determined for defective items under two-level trade credit policy in the EPQ framework. Four theorems for determining the optimal cycle time and the results in this study can be deduced as a special case of earlier models. Finally, illustrative examples are provided to verify the theoretical results.

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## 1. Introduction

The major goals in manufacturing environments are minimizing costs and improving overall quality. In fact, reducing inventory is the often the most effective way to control production costs. The classical economic order quantity (EOQ) model has been widely used by practitioners as an inventory control system. However, use of the model to find an economical lot size reveals several weaknesses based on unrealistic assumptions. One obvious example is that the model assumes that the customer pays the supplier immediately upon receiving the supplies whereas, in practice, suppliers often provide a grace period. Interest is rarely charged if the outstanding amount is paid within a specified grace period. However, during the trade credit period, the retailer earns interest on the payment received for the goods sold and thus accumulates revenue. Therefore, customers prefer to delay payment until the deadline given by the supplier. Inventory replenishment policies under trade credit have been studied intensively. Goyal (1985) established a single-item inventory model with a permissible delay in payments. Aggarwal and Jaggi (1995) extended the Goyal model to deteriorating items. Jamal, Sarker, and Wang (1997) and Chang and Dye (2001) further generalized the model to consider product shortages. Huang and Chung (2003), Chang and Teng (2004), Huang and Lin (2005) and Ouyang, Chang, and Teng (2005) proposed an optimal policy for a supplier offering both trade credit and cash discount. Chung and Huang (2003) and Liao (2007) mod-

ified the units to be replenished at a finite rate. Several studies, such as Chang, Ouyang, and Teng (2003), Chang (2004), Chung and Liao (2004, 2006) and Chung, Goyal, and Huang (2005), have considered how payment delays are affected by order quantity. Further, Huang (2003) first extended the Goyal model to develop an EOQ model in which the supplier offers the trade credit period  $M$  to the retailer, and the retailer in turn provides the trade credit period  $N$  to customers ( $N < M$ ). A follow-up study by Huang (2007) incorporated the studies of Chung and Huang (2003) and Huang (2003) to investigate optimal replenishment decisions of retailers under two-level trade credit policy within the economic production quantity (EPQ) framework, which reflects actual business practices. Chung and Huang (2007) developed an EOQ model for deteriorating items with limited storage capacity to determine the optimal ordering policy when the supplier provides a grace period to the retailer and the retailer adopts a trade credit policy to stimulate customer demand.

The classic EOQ model also implicitly assumes optimal and consistent quality, which is inconsistent with actual production environments. These defective items must be rejected, repaired, or reworked, and extra costs are thus incurred. Therefore, quality-related cost must be considered when developing policies for optimizing orders. Porteus (1986) refined the basic EOQ model by incorporating the effect of defective items. Rosenblatt and Lee (1986) proposed an EPQ model for a production system with an imperfect production process. Cheng (1989) developed an EPQ model with a flexible and imperfect process. Cheng (1991) proposed an EOQ model with demand-dependent unit production cost and imperfect production processes. Kim and Hong (1999)

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extended the Rosenblatt and Lee (1986) model by assuming an elapsed time until the shift is arbitrarily distributed. Salameh and Jaber (2000) developed an EOQ model in which a fraction of the ordered lot is of imperfect quality but uniformly distributed. Hayek and Salameh (2001) developed an EPQ model with uniformly distributed defects. Chiu (2003) extended the Hayek and Salameh (2001) model by assuming that a portion of the defective items are reworked to meet quality control standards and that the remaining items are sold at a sale price on the EPQ model with backlogging allowed. Huang (2002) developed a model for optimizing an integrated vendor–buyer inventory policy for flawed items in a just-in-time (JIT) manufacturing environment. Chan, Ibrahim, and Lochert (2003) provided a framework for integrating reduced prices, reworked products, and product rejection situations in a single EPQ model. Recently, Eroglu and Oadmir (2007) extended the model of Salameh and Jaber (2000) with allowable shortages and developed an EOQ model in which each ordered lot is screened to separate acceptable and defective items.

However, the infinite replenishment feature of the inventory model is inconsistent with actual industrial practice. Therefore, optimal ordering policy with finite replenishment rate has been studied intensively. Chung and Huang (2003) extended the Goyal (1985) model to the case in which the units are replenished at a finite rate. Huang and Lin (2005) investigated the optimal replenishment policy of retailers under permissible delay in payments and cash discount within the EPQ framework. Liao (2007) derived a production model for the lot-size inventory system with a finite production rate, which considered the effect of decay and the condition of a permissible payment delay.

None of the above models consider the effects of optimal replenishment decisions by the wholesaler under two-level trade credit policy with finite replenishment rate and imperfect product quality. This study extends existing EOQ models in three ways. First, it considers suppliers who offer a grace period to wholesalers as well as wholesalers who adopt a trade credit policy for the retailer. Secondly, as mentioned above, items of imperfect quality are not assumed to be defective since they may be satisfactory to other buyers or sold to other buyers at reduced prices. In this model, defective items are categorized as either imperfect items or scrap. Scrap items are discarded in a single batch after the screening process, and the imperfect items are sold as a single batch at the end of each cycle. This model further assumes that the screening rate is high, so item inspection is far faster than production. Hence, the end of the production period and the end of the screening process usually occur simultaneously. Thirdly, this study extends the EOQ model to the EPQ model by assuming a finite replenishment rate.

This study established a mathematical model with a finite replenishment rate to optimize ordering policies for defective items. Four theorems are derived to efficiently determine the optimal cycle time for wholesaler to maximize annual total profit. The results in this study can be considered a refinement of those published in earlier studies. Finally, illustrative examples are provided to verify the theoretical results.

## 2. Model formulation

This study uses the following notations and assumptions.

### Notation

$D$	demand rate per year
$P$	replenishment rate per year
$Q$	the order quantity
$A$	cost of placing an order
$c$	unit purchasing cost

$v$	unit selling price of imperfect items, $v < c$
$s$	unit selling price of good items
$p$	percentage of defective items in $Q$
$f(p)$	probability density function of $p$
$c_s$	unit disposal cost for scrap items
$h$	holding cost per unit per year excluding interest charges
$q$	percentage of scrap items in defective items
$d$	unit screening cost
$I_e$	interest earned per \$ per year
$I_k$	interest charged per \$ in stocks per year
$T$	the cycle time
$M$	wholesaler's trade credit period offered by the supplier in years
$N$	retailer's trade credit period offered by the wholesaler in years
$E(\cdot)$	expected value operator
$Z(T)$	annual total expected profit, which is a function of $T$
$T^*$	optimal cycle time of $Z(T)$

### Assumptions:

- (1) Demand rate,  $D$ , is known and constant.
- (2) Replenishment rate,  $P$ , is known and constant,  $P > D$
- (3) Shortages are not allowed.
- (4) Time horizon is infinite.
- (5) The sales revenue generated during the credit period is deposited in an interest-bearing account with rate  $I_k$ . At the end of the period, the trade credit is settled, and the buyer starts paying interest on the items in stock at rate  $I_e$ . ( $I_k \geq I_e$ )
- (6) The manufacturer trade credit period offered by supplier  $M$  is no shorter than the retailer trade credit period offered by manufacturer  $N$ . ( $M \geq N$ )
- (7) A 100% screening process has a rate of  $x$  units per unit time, and screening rate  $x$  is sufficiently large. This assumption is reasonable now that modern automatic screening machines and electronic control systems are widely used to quickly inspect all items produced or purchased.
- (8) The end of each production period occurs concurrently with the end of the screening process. For the reasons mentioned in assumption 7 above, the screening rate  $x$  is sufficiently large such that item inspection is performed faster than production. Under this condition, the production period and the screening process are expected to end simultaneously.
- (9) Each lot contains a percentage of defective items of,  $p$ , with a known probability density function,  $f(p)$ . The defective items comprise reworkable items  $[(1 - q) \cdot p]$  and scrap items  $(q \cdot p)$ . The scrap items must be removed from inventory at a unit cost,  $c_s$ , at the end of the screening process. Reworkable items are sold in a single batch at a discounted price,  $v$ , per unit at the end of the cycle ( $c > v > c_s$ ).

### The model:

Let  $N(Q, p)$  denote the number of non-defective items in each lot. As Fig. 1 shows, some of these good items meet the demand with a rate of  $D$  in the cycle time of  $T$ , which are:

$$N(Q, p) = Q - pQ = DT \quad (1)$$

To avoid shortages, the number of acceptable items is assumed to large the demand of the production period, that is:

$$N(Q, p) \geq Dt \quad (2)$$

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