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Credit rating dynamics in the presence of unknown structural breaks

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ABSTRACT

In many credit risk and pricing applications, credit transition matrix is modeled by a constant transition probability or generator matrix for Markov processes. Based on empirical evidence, we model rating transition processes as piecewise homogeneous Markov chains with unobserved structural breaks. The proposed model provides explicit formulas for the posterior distribution of the time-varying rating transition generator matrices, the probability of structural break at each period and prediction of transition matrices in the presence of possible structural breaks. Estimating the model by credit rating history, we show that the structural break in rating transitions can be captured by the proposed model. We also show that structural breaks in rating dynamics are different for different industries. We then compare the prediction performance of the proposed and time-homogeneous Markov chain models.

1. Introduction

In modern credit risk management, it is convenient to assume that the credit quality (or rating) of obligors (or firms) follows a time homogeneous Markov chain which is characterized by a credit rating transition matrix. Since the credit rating transition matrix summarizes historical changes in credit rating of obligors, it has wide applications in finance. In pricing models for bond and credit derivatives, the valuation of risky credit derivatives is based on the credit ratings of obligors (Jarrow et al., 1997, 1998; Lando, 2000; Acharya et al., 2006). In portfolio risk assessment, the risk is measured by the joint distribution of rating transitions for the loans and bonds which make up the portfolios of interest (Das et al., 2006; Frey and McNeil, 2007; Egloff et al., 2007; Duffie et al., 2009; Tsaig et al., 2011). In the Basel Accord for bank regulation, banks are required to construct credit rating transition matrices based on their own data to stress test their portfolios and evaluate evidence of rating transitions in external ratings (Treacy and Carey, 2000; Altman and Rijken, 2004; Gordy and Howells, 2006). In the credit rating industry, rating agencies such as Moody's, Standard and Poor, and Fitch publish reports on rating transitions for obligors or financial instruments, which are studied by credit risk managers, and some risk management tools such as Morgan's CreditMetrics are built on estimates of rating transition matrices.

The estimates of credit rating transition matrices published by rating agencies usually use a discrete-time setting. However, due to the availability of rating data and the well known advantages of using the continuous time Markov approach over the discrete one (Lando and Skødeberg, 2002), a continuous time homogeneous Markov framework is usually assumed for the rating process. In particular, suppose that there are K rating categories and the rating migration process of a firm for the period $(0,t)$ is a continuous time homogeneous Markov chain with transition matrix $P(0,t)$, in which the ijth entry represents the probability of migrating from category i to category j during the period $(0,t)$. Similar to the discrete-time Markov process for which the rating transition matrix can be obtained by matrix multiplication from the one-period transition matrix, the matrix $P(0,t)$ can be represented, under the assumption of time homogeneity, through its generator matrix Λ, that is, for any time $t > 0$,

$$P(0,t) = \exp(\Lambda t) := \sum_{k=0}^{\infty} \frac{\Lambda^k t^k}{k!}, \tag{1}$$

in which $\Lambda = (\lambda^{(i,j)})$ satisfies $\lambda^{(i,i)} = -\sum_{j\neq i}\lambda^{(i,j)}$ for $1 \leqslant i \leqslant K$, and $\lambda^{(i,j)} \geqslant 0$ for $1 \leqslant i \neq j \leqslant K$. The elements in Λ can be estimated by their maximum likelihood estimators $\hat{\lambda}^{(i,j)} = \widetilde{N}_{ij}/\int_0^t Y_i(s)ds$, in which $Y_i(s)$ is the number of firms in rating class i at time s and $\widetilde{N}_{ij}$ is the total number of transitions from i to $j(\neq i)$ over the period $(0,t)$ (Küchler and Sørensen, 1997, p. 26).

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However, the assumptions of time homogeneity and Markovian behavior of the rating process in (1) have been challenged by recent studies on the presence of various non-Markovian behaviors such as industry heterogeneity, ratings drift, and time variations; see details in Altman (1998), Blume et al. (1998), Nickell et al. (2000), Bangia et al. (2002), Christensen et al. (2004) and Liao et al. (2009) and reference therein. Since the ratings are sometimes considered to ''look through the cycle'', firms' ratings could be remarkably stable over credit cycles; see Fons (2002), Cantor and Mann (2003a,b), Altman and Rijken (2004) and Bruche and González-Aguado (2010). To address this issue without using firm-specific information, Frydman and Schuermann (2008) consider the behavior that firms with the same rating migrate at different speeds and propose a mixture model of two independent continuous time homogeneous Markov chains for the rating migration process. They apply the proposed model to analyze corporate credit rating history from Standard & Poor's spanning 1981–2002.

This paper studies the variations in rating transition matrices and proposes a hidden Markov model which extends the continuous time homogeneous Markov chain and characterizes unobserved structural breaks in the credit rating dynamics. From a credit risk modeling perspective, it is important to study variations in rating transitions attributable to the state of the economy. Some credit risk modeling approaches classify the states of the economy as finite regimes and assume transition matrices change over time with the states of the economy. This ignores the fact that the states of the economy at different periods might be different even they are in the same regime. From this perspective, it would be better not to restrict the number of regimes. We hence assume the state of the economy to be continuous, and model the structural changes in the economy as the shifts of the state of the economy in a continuous space.

Motivated by the discussion above, we propose a stochastic structural break model for rating migration processes. Different from Crowder et al. (2005) who model the occurrence of defaults within a bond portfolio as hidden Markov process, we assume that, in our model, the generators of the rating transition matrices are constant between two adjacent structural changes in the economy. As the states of the economy may have infinite regimes, the generators are assumed to follow a continuous state and continuous time nonhomogeneous hidden Markov process, and be piecewise constant with the number, time and magnitude of the structural changes unobserved. We further model the structural breaks in the generators as a compounded Poisson process, in which the times of structural breaks follow a Poisson process with a constant rate and the entries of post-change generator matrices follow a Gamma distribution. These assumptions allow us to derive the distributions of the time-varying generators of rating migration matrices and the probability of structural breaks at each time period, given firms' transition history. The derived distribution of generator matrices at a given period is a mixture of Gamma distributions, and the weights of mixture components can be computed explicitly using historical observations. As the number of mixture components changes over time, the model is allowed to incorporate various non-Markovian behaviors in empirical studies. From this perspective, our model extends the mixture model of two independent continuous time homogeneous Markov chains in Frydman and Schuermann (2008). The proposed model also implies a prediction formula for generator matrices with probable structural breaks in the future, for which the intensity or probability of structural breaks in generator matrices should be calibrated by historical observations.

We use the proposed model and developed inference procedure to study the monthly corporate credit ratings provided by Standard & Poor's from January 1985 to September 2009. We show that the generator and transition matrices of rating transitions are indeed changing over time, and the estimated structural breaks are not only statistically significant but economically meaningful as well. The estimated times of structural breaks match the times of several significant structural changes in the economy. We also demonstrate that the generator or transition matrices in different industry categories have different behaviors, and specifically, industry sectors related to finance services are more susceptible to economic changes than other sectors. We further conduct out-of-sample forecast evaluations of our model against the time homogeneous Markov model and compare the performance of our model with that of a time-homogeneous Markov model. The comparison shows that our model provides a more accurate prediction for rating generator or probability matrices.

The remainder of the paper is organized as follows. Section 2 develops a stochastic structural break model for generator matrices and its inference procedure using continuous credit rating history. In Section 3, we study the in-sample and out-of-sample performance of our model on the data set, and discuss the estimation results and their economic implications. Section 4 provides some concluding remarks.

2. Stochastic structural break model

We assume that a rating transition process of an obligor follows a K-state non-homogeneous continuous time Markov process. This process is further characterized by a transition probability matrix $P(s,t)$ over the period (s,t), in which the ijth element of $P(s,t)$ represents the probability that an obligor starting in state i at time s is in state j at time t. Suppose that there are n rating transitions observed over the period (s,t). For a transition time t_i in (s,t), denote $\Delta N_{kj}(t_i)$ the number of transitions observed from state k to state j at time t_i, $\Delta N_k(t_i) = \sum_{1\leqslant j\leqslant K, j\neq k}\Delta N_{kj}(t_i)$, and $Y_k(t_i)$ the number of firms in state k right before time t_i. The transition matrix $P(s,t)$ can be consistently estimated by the product-limit estimator

$$\widehat{P}(s,t) = \prod_{i=1}^{n}(I + \Delta\widehat{A}(t_i)),$$

in which

$$\Delta\widehat{A}(t_i) = \begin{pmatrix} -\frac{\Delta N_1(t_i)}{Y_1(t_i)} & \frac{\Delta N_{12}(t_i)}{Y_1(t_i)} & \frac{\Delta N_{13}(t_i)}{Y_1(t_i)} & \cdots & \frac{\Delta N_{1K}(t_i)}{Y_1(t_i)} \\ \frac{\Delta N_{21}(t_i)}{Y_2(t_i)} & -\frac{\Delta N_2(t_i)}{Y_2(t_i)} & \frac{\Delta N_{23}(t_i)}{Y_2(t_i)} & \cdots & \frac{\Delta N_{2K}(t_i)}{Y_2(t_i)} \\ \vdots & \vdots & & \cdots & \vdots \\ \frac{\Delta N_{K-1,1}(t_i)}{Y_{K-1}(t_i)} & \frac{\Delta N_{K-1,2}(t_i)}{Y_{K-1}(t_i)} & \cdots & -\frac{\Delta N_{K-1}(t_i)}{Y_{K-1}(t_i)} & \frac{\Delta N_{K-1,K}(t_i)}{Y_{K-1}(t_i)} \\ 0 & 0 & \cdots & \cdots & 0 \end{pmatrix},$$

see Andersen et al. (1995, Section IV.4). In the matrix above, the kth diagonal element counts the fraction of the exposed firms $Y_k(t_i)$ leaving the state at time t_i, and the kjth off-diagonal element count the fraction of transitions from the kth category to the jth category in the number of exposed firms at time t_i. Note that the variable Y has incorporated the case of censoring for which there is no change in the estimator at the time of a censoring event. Furthermore, the last row in $\Delta\widehat{A}(t_i)$ is zero because the kth state (i.e., default state) is absorbent. Since the purpose of our model is to incorporate structural changes into credit rating dynamics, we assume from now on that the non-homogeneous continuous time Markov process can be decomposed as piecewise homogeneous continuous time Markov processes with unobserved structural breaks.

2.1. Model specification

Specifically, we assume that the structural breaks in credit rating generator matrices follow a Poisson process $\{N_{\mathcal{A}}(t); t \geqslant 0\}$ with constant rate η, hence the duration between two adjacent