



Reinforced urn processes for credit risk models



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ABSTRACT

We propose a Bayesian nonparametric model to estimate rating migration matrices and default probabilities using the reinforced urn processes (RUP) introduced in Muliere et al. (2000). The estimated default probability becomes our prior information in a parametric model for the prediction of the number of bankruptcies, with the only assumption of exchangeability within rating classes. The Polya urn construction of the transition matrix justifies a Beta distributed de Finetti measure. Dependence among the processes is introduced through the dependence among the default probabilities, with the Bivariate Beta Distribution proposed in Olkin and Liu (2003) and its multivariate generalization.

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1. Introduction

About 30 years ago most financial institutions relied on subjective analyses to assess the credit risk of corporate loans, with expert bankers who used borrower information, such as reputation, leverage, earnings history and collaterals, to reach a largely subjective judgement (Altman and Saunders, 1998). Naturally, over the years, banks have moved away from these systems towards more objective criteria. One of these methods is the so called credit-scoring systems, based on accounting ratios of potential borrowers, see Altman and Narayanan (1997), Altman et al. (1977) and Platt and Platt (1991). Still, subjective assessment of default probabilities can be approached at a higher level of formalism, thanks to the Bayesian approach: see for instance Kiefer (2011), where expert opinions are included as prior information on the default probability, with a maximum entropy approach that minimizes the information beyond the expert opinion. Alternative objective approaches are the

“risk of ruin” models: a firm goes bankrupt when the market value of its assets A falls below its debt obligations D (Wilcox, 1973; Scott, 1981; Santomero and Vinso, 1977), following the idea of the option pricing models of Black and Scholes (1973) and Merton (1974). Similarly, thresholds for $A - D$ can be established for transitions to other rating states (Bangia et al., 2002). A second, newer class of models are those that derive implied probabilities of default from the term structure of the yield spreads between default-free and risky securities: see, among others, Iben and Litterman (1989). Finally, there are capital market based models, currently adopted by the main rating agencies, that estimate probabilities of defaults from past default data. See, for example, Altman (1989) and Asquith et al. (1989). To this class belongs the estimation method for rating migration matrices which has become the industry standard nowadays: the straightforward *Cohort Approach*.

We aim at introducing a new model for the estimation of rating migration matrices, through a Bayesian nonparametric approach, using the Reinforced Urn Processes (RUP) introduced in Muliere et al. (2000). The idea is to model the rating history of each firm as a process that can only assume a finite number of states, from 0 (the highest rating) to L (default). We will assume time to be discrete, and thanks to Polya reinforcement mechanisms will be able to learn from history in the estimation of the matrices. Being based on capital market data, our model is not affected by the main limit

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of credit-scoring models, that is their failure to capture and include most of the fast-moving changes in borrower conditions. Furthermore, the Cohort Approach estimates the probability of moving from class i to j as the simple proportion of firms with rating j at the end of the period, having started at rating i . Obviously, this can imply a relevant loss of information since any change occurring within the period is ignored. With our methodology, changes occurring within the period of study contribute to the estimation: for example, even if no default was observed in the sampling period for rating class 'AAA', the probability of default for this class can still be greater than 0 if some 'AAA'-rated firms moved to, say, 'A-', and some defaults in the class 'A-' were observed. Finally our model can be easily extended to default probabilities depending on the number of defaults occurred before: this can be particularly useful for instance when there is a concrete risk of contagion among the firms, with the default probability increasing with the number of observed default.

In the second part of the paper, we use the estimated migration matrix, and in particular the estimated default probability, as prior information for a parametric model for the prediction of bankruptcies. Formally, the number of bankruptcies in different sectors and rating classes is modelled as multiple partially exchangeable \mathbb{N} -valued stochastic processes with positive jumps of size 1, that is as counting processes $\{X_t\}_{t \geq 0}$. X_t counts the number of defaults which has occurred between time 0 and t . The Polya urn construction of the transition matrix justifies a Beta distribution for the default probability of the process, with average equal to the probability of default estimated through the proposed nonparametric procedure. We study the joint behaviour of the bankruptcies among sectors and rating classes, with the only assumption of exchangeability within rating classes. Under this assumption and with one single process, the finite dimensional distribution of the number of bankruptcies is the well known Beta-Binomial distribution. We extend this setting to several processes and beyond exchangeability, that is for several sectors and rating classes. Dependence among the processes is introduced through dependence among the default probabilities, with the Bivariate Beta Distribution proposed in Olkin and Liu (2003) and its multivariate generalization.

Strictly related to the procedure adopted in this paper is the multivariate counting process of Andersen et al. (1985): in our setting, their hazard rate depends, in each rating class, on the current composition of the Polya urns and is reinforced through time by past defaults and rating transitions. Among other recent developments of Bayesian nonparametrics in econometrics, Hirano (2002) proposes a semiparametric first-order autoregressive model with random effects, where the innovations are represented as the Dirichlet Process Mixtures introduced in Lo (1984). Taddy and Kottas (2009) introduced a first-order hidden Markov switching model with both observations and regressors jointly characterized as Dirichlet Process Mixtures, and with dependence introduced through the state membership, which constitutes a hidden stationary Markov chain. They also elaborate a Forward-Backward Gibbs sampler for efficient posterior simulation. Along the same line, Jensen and Maheu (2010) generalizes stochastic volatility models to return innovations sampled from a Dirichlet Process Mixture distribution, and Griffin (2011) model volatility as a countable mixture of Non-Gaussian Ornstein-Uhlenbeck Processes, mixed by exchangeable latent parameters from a Dirichlet Process. Finally, Kalli et al. (2013) extend GARCH-type models to nonparametric multiplicative innovations from a stick-breaking mixture of uniform densities.

In Section 2 the Polya Urn-based method for the estimation of the rating transition matrix is introduced, while the model for bankruptcies is proposed in Section 3. In Section 4 an empirical application illustrates our methodology and we conclude in Section 5.

2. A Polya urn model for rating migrations

2.1. Polya urn models and reinforced urn processes

Urn models have a long and relevant tradition in probability and statistics. The urn scheme proposed by Eggenberger and Polya (1923) and Polya (1930) was introduced for describing contagious diseases and became soon the prototype for many probabilistic models used in different fields of applications. Consider an urn initially containing $m_0(b) \geq 0$ black balls and $m_0(w) \geq 0$ white balls. At stage $t = 1, 2, \dots$ we sample a ball from the urn and modify the composition of the urn following certain specific rules which characterize the urn process. This generates a sequence of random variables $\{X_t\}$, each equal to 0 or 1 according to the colour, white or black respectively, of the ball sampled at stage t . Moreover, for all $t \geq 1$, let B_t and W_t be the number of black and white balls in the urn before the $t + 1$ th stage. Define Y_t as the proportion of black balls contained in the urn before the $t + 1$ th stage, that is $Y_t = B_t / (B_t + W_t)$. In the original paper of Eggenberger and Polya (1923), at each stage $t \geq 1$, a ball is chosen at random from the urn and put back into it along with other $s \geq 1$ balls of the same colour. Then,

$$X_t \sim \text{Bern} \left(\frac{m_0(b)}{m_0(b) + m_0(w)} \right).$$

For all $t \geq 1$, conditionally on X_1, \dots, X_t , we have $X_{t+1} \sim \text{Bern}(Y_t)$, where, starting with $(B_0, W_0) = (m_0(b), m_0(w))$,

$$(B_{t+1}, W_{t+1}) = \begin{cases} (B_t, W_t + s) & \text{with probability } 1 - Y_t \\ (B_t + s, W_t) & \text{with probability } Y_t. \end{cases}$$

Coppersmith and Diaconis (unpublished) introduced the idea of reinforced random walks, for modelling situations where a random walker has a tendency to revisit familiar territory (see also Diaconis, 1988; Pemantle, 1988). Reinforced random walks and Polya urns find a common ground in Muliere et al. (2000) where the Reinforced Urn Process (RUP) is introduced: a reinforced random walk on a countable state space of Polya urns. Under suitable conditions, such a process, say $\{Z_t\}$ with elements in the space V , can be characterized by a mixture of Markov chains (Diaconis and Freedman, 1980), with ready applications to survival data or, more generally, to multiple state processes assumed to be conditionally independent Markov chains. More formally, this means that there exists a probability distribution μ on the set \mathcal{P} of stochastic matrices on $V \times V$ such that, for all $t \geq 1$ and all finite sequences (z_1, \dots, z_t) of elements of V ,

$$\mathbb{P}(Z_0 = z_0, \dots, Z_t = z_t) = \int_{\mathcal{P}} \prod_{j=0}^{t-1} \pi(z_j, z_{j+1}) \mu(d\pi).$$

Fortini et al. (2002), formalizing the idea in de Finetti (1959), characterize mixtures of recurrent Markov chains in terms of row exchangeability (partial exchangeability in the sense of de Finetti) of the matrix of successor states, that is the matrix whose (i, j) -element denotes the value of the stochastic process immediately after the j th visit to state i . This result is extended in Epifani et al. (2002) to mixtures of recurrent semi-Markov processes, characterized in terms of row exchangeability of the matrix of successor states and holding times. Furthermore, Muliere et al. (2003) extended the analysis of reinforced random processes to continuous time. The RUP specification depends on four elements: a countable state space, V ; a finite set of colours, C , of cardinality $h \geq 1$; an urn composition function, U , which maps V into the set of h -tuples of nonnegative real numbers whose sum is a strictly positive number; a law of motion, $d : V \times C \rightarrow V$. To each $v \in V$ it is associated an urn $U(v)$, which initially contains $m_v(c) \geq 0$ balls of colour c , for each $c \in C$. Without loss of generality, it is assumed that the law of motion d has the property that, for every

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