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Inventory models for deteriorating items with maximum lifetime under downstream partial trade credits to credit-risk customers by discounted cash-flow analysis

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ABSTRACT

Getting loans from banks are almost impossible after 2008 global financial crisis. As a result, about 80% of companies in United Kingdom and United States offer their products on various short-term, free-interest loans (i.e., trade credit) to customers. Numerous researchers and academicians apply discounted cash flow (DCF) analysis merely to compute the interest earned and charged during the credit period but not to the revenue and other costs which are considerably larger than the interest earned and charged. For a rigorous analysis, the DCF on all relevant costs is applied. In addition, many products deteriorate continuously and cannot be sold after their maximum lifetimes or expiration dates. However, very few researchers and investigators have implemented the product lifetime expectance into their models. In this paper, a supplier–retailer–customer chain system is developed in which the retailer gets an upstream full trade credit from the supplier whereas offers a downstream partial trade credit to credit-risk customers, the deterioration rate is non-decreasing over time and near 100% particularly close to its expiration date, and DCF analysis is applied to compute all relevant costs. This paper demonstrates that the retailer's optimal replenishment cycle time not only exists but also is unique. Thus, the search of the global optimal reduces to finding a local solution. Finally, several numerical examples and sensitivity analysis are performed in order to illustrate the problem and obtain managerial insights.

1. Introduction

In traditional business transactions, it was implicitly assumed that the buyer must pay the procurement cost when products are received. However, in today's competitive markets, most companies offer buyers various credit terms (e.g., permissible delay in payments, cash discounts, etc.) to stimulate sales and hence reduce inventory. Seifert et al. (2013) stipulated that "estimates suggest that more than 80% of business-to-business transactions in the United Kingdom are made on credit, while about 80% of the United States firms offer their products on trade credit." Conversely, it is known that trade credit decreases the buyer's inventory holding cost, and therefore affects the buyer's economic order quantity (thereafter, EOQ).

In review of the literature for inventory models with trade credit financing, Goyal (1985) derived the retailer's optimal EOQ when the supplier provides a permissible delay in payment (i.e., an upstream trade credit). Aggarwal and Jaggi (1995) extended the EOQ model for non-deteriorating items (i.e., products can be stored indefinitely) to deteriorating items (i.e., products deteriorate constantly). Jamal et al. (1997) further expanded the model to permit shortages. Teng (2002) modified the previous models by using sales revenue (instead of purchase amount) to compute the interest earned from sales. Huang (2003) then explored the problem to a supply chain system in which the retailer receives an upstream trade credit from the supplier while offers a downstream trade credit to customers. Liao (2008) further generalized Huang's model with an unlimited replenishment rate to an economic production quantity (thereafter, EPQ) model with a limited replenishment rate for deteriorating items. Min et al. (2010) proposed an EPQ model with both upstream and downstream trade credits when the demand depends on on-hand displayed stocks. Teng et al. (2012) extended the constant demand rate to the increasing demand pattern for the growth stage of product life cycle. Chern et al. (2013, 2014) discussed non-cooperative Stackelberg and Nash two-player equilibrium solutions between the

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supplier and the retailer. Chen et al. (2014a, 2014b) discussed the retailer's optimal EOQ/EPQ under different credit terms. Liao et al. (2014) studied optimal strategy for deteriorating items with capacity constraints under upstream and downstream trade credits.

Various products such as volatile liquids, blood banks, vegetables, fruits, fashion merchandises and high-tech products deteriorate continuously due to evaporation, spoilage, obsolescence, among other reasons. Ghare and Schrader (1963) built an inventory model with an exponentially decaying inventory. Covert and Philip (1973) extended the constant deterioration rate to Weibull failure rate. Dave and Patel (1981) proposed an EOQ model for deteriorating items with linearly increasing demand. Sachan (1984) further generalized the EOQ model to permit shortages. Hariga (1996) derived EOQ models for deteriorating items when the demand function is log-concave with time (e.g., a constant demand or an increasing demand pattern is a special case). Teng et al. (1999) further expanded EOQ models to allow for shortages and any continuous demand pattern (which includes a log-concave demand as a special case). Teng et al. (2002) further explored the model to permit partial backlogging. Dye (2013) investigated the effect of technology investment on refrigeration to improve the profit for deteriorating products. It is important to mention that none of the above cited papers took the expiration date into consideration until studies such as in Chen and Teng (2014), Sarkar (2012), Wang et al. (2014), Wu et al. (2014) and Sarkar et al. (2015).

There were some research works related to DCF analysis. For example, Hill and Pakkala (2005) applied DCF analysis to determine the optimal inventory policy that maximizes the expected net present value of the cash flows. Chung and Liao (2006) using DCF developed an inventory model for deteriorating items with permissible delay in payments. Dye et al. (2007) took pricing into consideration and derived the optimal inventory and pricing strategies to maximize the net present value of total profit. Chang et al. (2010) established the optimal ordering policies for deteriorating items using DCF analysis when the seller offers a trade credit if the buyer orders more than or equal to the predetermined amount. Mousavi et al. (2013) used meta-heuristic algorithms to solve the multi-item and multi-period inventory systems with DCF. Recently, Chen and Teng (2015) applied the DCF analysis to obtain the optimal lot size and credit period in a supply chain with upstream and downstream trade credit financing.

In reality, a credit-worthy retailer generally obtains a permissible delay on the entire purchasing quantity without collateral deposits from the supplier (i.e., an upstream full trade credit). However, a retailer often asks for credit-risk customers to cover a fraction of the purchasing cost as a collateral deposit at the time of placing an order, and then provides a permissible delay on the remaining quantity (i.e., a downstream partial trade credit). The use of a downstream partial trade credit as a strategy to reduce default (or bad debt) risks with credit-risk customers has received relatively little attention by the researchers and investigators. Additionally, the majority of the recent studies considers merely the opportunity loss (i.e., time value of money) of trade credit but ignores to take the opportunity loss of the other costs into consideration. For a rigorous and sound analysis, the DCF analysis must be used in order to take the effect of inflation and time value of money on all relevant revenue and costs. As a result, in this paper, a supplier–retailer–customer supply chain system is proposed in which the supplier provides the retailer an upstream full credit period of S years, meanwhile the retailer in turn gives a downstream partial credit period of R years to the customer, the deteriorating rate is constant or increasing and closer to 100% especially near to its expiration date, and the DCF analysis is applied to incorporate the effects of inflation and time value of money. In fact, the most commonly used Weibull non-decreasing deterioration (or failure) rate is a special case of the proposed generalized deterioration rate. This paper formulates the retailer's objective functions under different possible alternatives. By applying existing theorems in concave functions, it is proved that there exists a unique global optimal solution to each alternative. Finally, some numerical examples are solved in order to illustrate the problem and also some managerial insights are given.

The remaining of the paper is designed as follows. Section 2 defines notation and makes necessary assumptions. Section 3 derives the present value of the retailer's annual total profit under each alternative. Section 4 shows that the optimal cycle time exists uniquely which simplifies the search for the global optimal to a local one. Section 5 presents numerical examples in order to illustrate the model, and obtain managerial insights. Finally, Section 6 provides the conclusions and the future research directions.

2. Notation and assumptions

The following notation and assumptions are used in the entire paper.

2.1. Notation

For simplicity, the symbols for parameters, decision variable, and functions are defined accordingly.

Parameters

α	fraction of the purchasing cost must be paid at the time of placing an order, $0 \le \alpha \le 1$
c	purchasing cost per unit in dollars
D	demand rate in units per year
h	holding cost per unit per year in dollars excluding interest charge
i	annual compound interest per dollar per year
m	maximum lifetime or expiration time in years
o	ordering cost per order in dollars
p	selling price per unit in dollars
R	downstream credit period in years by the retailer
S	upstream credit period in years by the supplier
t	time in years
I_c	interest charged per dollar per year
I_e	interest earned per dollar per year

Decision variable

T	replenishment cycle time in years

Functions

Q	oder quantity in units per replenishment cycle, which is a function of T
$\theta(t)$	deterioration rate at time t, which is a non-decreasing function in t
I(t)	inventory level at time t
$PTP(T)$	present value of annual total profit, which is a function of T

For convenience, the asterisk symbol on a variable is denoted the optimal solution of the variable. For instance, T^* is the optimal solution of T. Next, the necessary assumptions to build up the mathematical model are given below.