Insurance: Mathematics and Economics 51 (2012) 567–575

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

On a reduced form credit risk model with common shock and regime switching

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ARTICLE INFO

Article history:
Received March 2012
Received in revised form
July 2012
Accepted 30 July 2012

Keywords:
Common shock
Markov chain
Cox process
Regime switching
Correlated defaults
Basket default swaps

ABSTRACT

Reduced form credit risk models are important ones in credit risk theory. In such a model, certain correlated relations are constructed to represent the default dependence structure among the default intensity processes. In this paper, we introduced a reduced form credit risk model in which the default dependence structures among default intensity processes are described by the so-called common shocks with regime-switching. We derive some closed-form expressions for the joint distribution of the default times and for the pricing formulas of the basket default swaps. We also give numerical results to show the applicable aspects of the proposed model.

1. Introduction

Despite the occurrence of the 2008 financial crisis, credit derivatives remain the most popular tools to transfer and to hedge credit risk. The pricing of credit derivatives and the credit risk management applications require reasonable models for default risk, especially for correlated default risk. Reduced form models are widely used for this purpose. Within this class of model, there are four major approaches for generating default dependence: the copula models such as Schonbucher and Schubert (2001), Hull and White (2004), the interacting intensities models such as Davis and Lo (2001), Yu (2007), the conditionally independent default models such as Duffie and Garleanu (2001), and the common shock models such as Lindskog and McNeil (2003). In the copula models, the dependence structure is constructed by a copula function. In the interacting intensity models, the dependence between defaults is introduced via the interactions between default intensities. In the conditionally independent default models, default dependence comes from dependent default intensities but conditionally independent on a common set of state variables. In the common shock models, the defaults are related to a series of independent shock processes. McNeil et al. (2005) provide a comprehensive introduction to these models.

In this paper, we propose conditionally independent default models in which the dependence structure among the default intensity processes is constructed via the so called common shock with regime switching. The default times are defined as the first jump times of jump processes with exogenous intensities (see, e.g., Jarrow and Turnbull (1995), Lando (1998), Jarrow and Yu (2001), Duffie and Garleanu (2001), Yu (2007)). In the proposed reduced form credit risk model, defaults are driven by exogenous events such as policy events, liquidity events, natural catastrophes events, etc, whose arrivals are described by independent counting processes. These stochastic events are classified into several groups, and it is assumed that each event occurring at some time in one of these groups may lead to defaults in some defaultable firms with certain probabilities. We thus obtain the common shock dependence structure to generate the dependence between the default intensity processes, which is similar to that of Wang and Yuen (2005), who produce a certain correlation between claim-number processes for a correlated insurance risk model.

Since the macro-economics state creates dependent financial risks, regime switching approaches are widely used by many authors to capture correlated financial risks. See, for example, Buffngton and Elliott (2002), Elliott et al. (2005), Zhu and Yang (2008), Chen et al. (2008), Chen and Yang (2010), Capponi and Figueroa-López (2011). In this paper, we use a continuous-time observable Markov chain to represent the switches among the different economic states.

In Section 2, we introduce the reduced form credit risk model with the dependence structure of common shock, but without regime switching. We derive the joint distribution of default times under the proposed framework. In Section 3, we extend the study of Section 2 to the case with regime switching. In Section 4, we use

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doi:10.1016/j.insmatheco.2012.07.010

the results given in the previous two sections to price the basket default swaps. We get the closed form expression for the pricing formula. We present numerical example to show the applicable aspects of the results of this paper in Section 5.

2. The model without regime switching

Suppose that there are n firms concerned, and that there are m groups of stochastic events that may cause the n firms to default. Denote by $N^k(t)$ the number of the stochastic events from the kth group occurred up to time t. We assume that each event occurring at time t in the kth group may lead to the default of firm i with probability $p_i^k(t)$ and that whether an event in the kth group gives rise to a default of firm i or not is independent of all other events for $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$. For convenience, we mathematically denote by $N_i^k(t)$ the number of default times occurring in $[0, t]$ reduced from the stochastic events in group k.

Assumption 2.1. The processes $\{N^1(t)\}, \{N^2(t)\}, \ldots, \{N^m(t)\}$ are assumed to be independent Poisson processes with parameters $\lambda_1, \lambda_2, \ldots, \lambda_m$, respectively.

Assumption 2.2. For each k, the processes $\{N_1^k(t)\}, \{N_2^k(t)\}, \ldots, \{N_n^k(t)\}$ are assumed to be conditionally independent given $N^k(t)$.

Under Assumptions 2.1 and 2.2, we see that $\{N_i^k(t)\}$ is the $p_i^k(t)$-thinning of Poisson process $N^k(t)$ and, thus, it is a non homogeneous Poisson process with intensity process $\{\lambda_k p_i^k(t)\}$. We set

$$N_i(t) = \sum_{k=1}^{m} N_i^k(t), \quad i = 1, 2, \ldots, n. \tag{1}$$

Obviously, $N_i(t)$ represents the number of default times of firm i occurring in $[0, t]$. Note that the use of $N_i(t)$'s does not bring any confusion in defining the default times of the firms concerned. Using process $\{N_i(t)\}$, we see that the default time, τ_i, of firm i is, in fact,

$$\tau_i = \inf\{t \geq 0 : N_i(t) > 0\}, \quad i = 1, 2, \ldots, n. \tag{2}$$

Remark 2.1. Under Assumptions 2.1 and 2.2 and from Wang and Yuen (2005) we get that

1. The m vector processes $\{(N^1(t), N_1^1(t), N_2^1(t), \ldots, N_n^1(t))\}$, $\{(N^2(t), N_1^2(t), N_2^2(t), \ldots, N_n^2(t))\}, \ldots, \{(N^m(t), N_1^m(t), N_2^m(t), \ldots, N_n^m(t))\}$ are independent.
2. For $i = 1, 2, \ldots, m$, the Process $\{N_i(t)\}$ is a non homogeneous Poisson process with intensity $\sum_{k=1}^m \lambda_k p_i^k(t)$.

Remark 2.2. A similar reduced form credit risk model to (1) and (2) is considered in Giesecke (2003).

The goal of this section is mainly to derive the joint distribution of the default times. The following two lemmas are used for this purpose. Using the fine properties of Poisson process, one can verify that the following Lemma 2.1 holds true.

Lemma 2.1. *Suppose that $N(t)$ is a Poisson process with parameter λ. Let $0 < t_1 < t_2, l_1 \leq l_2, l_1, l_2 \in \{0, 1, 2, \ldots\}$, then we have*

$$P(N(t_1) = l_1 \mid N(t_2) = l_2) = C_{l_2}^{l_1}\left(\frac{t_1}{t_2}\right)^{l_1}\left(1 - \frac{t_1}{t_2}\right)^{l_2 - l_1}.$$

Using Lemma 2.1 and following the idea in the proof of Proposition 2.3.2 of Ross (1996), one can verify that the following Lemma 2.2 holds true.

Lemma 2.2. *Let $0 < t_1 < t_2, l_1 \leq l_2, l_1, l_2 \in \{0, 1, 2, \ldots\}$, we have*

$$\begin{aligned} &P(N_j^k(t_1) = l_1 \mid N^k(t_2) = l_2) \\ &\quad = C_{l_2}^{l_1}\left(\frac{\beta_j^k(t_1)}{t_2}\right)^{l_1}\left(1 - \frac{\beta_j^k(t_1)}{t_2}\right)^{l_2 - l_1}, \\ &\qquad k = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n, \end{aligned}$$

where

$$\beta_j^k(t_1) = \int_0^{t_1} p_j^k(s)ds.$$

Proposition 2.1. *Let $i \neq j,\ i, j \in \{1, 2, \ldots, n\}$, and $0 < t_1 < t_2 < \cdots < t_n$, then we have*

$$P(\tau_i > t_1, \tau_j > t_2) = e^{-\sum_{k=1}^m \lambda_k t_2\left[1-\left(1-\frac{\beta_i^k(t_1)}{t_2}\right)\left(1-\frac{\beta_j^k(t_2)}{t_2}\right)\right]},$$

and

$$\begin{aligned} &P(\tau_1 > t_1, \tau_2 > t_2, \ldots, \tau_n > t_n) \\ &\quad = e^{-\sum_{k=1}^m \lambda_k t_n\left(1-\prod_{j=1}^n\left(1-\frac{\beta_j^k(t_j)}{t_n}\right)\right)}. \end{aligned} \tag{3}$$

Proof. From (1) and (2), we have

$$\begin{aligned} P(\tau_i > t_1, \tau_j > t_2) &= P(N_i(t_1) = 0, N_j(t_2) = 0) \\ &= P(N_i^1(t_1) = 0, \ldots, N_i^m(t_1) = 0, \\ &\quad N_j^1(t_2) = 0, \ldots, N_j^m(t_2) = 0). \end{aligned}$$

By the first statement of Remark 2.1, we have

$$\begin{aligned} &P(N_i^1(t_1) = 0, \ldots, N_i^m(t_1) = 0, N_j^1(t_2) = 0, \ldots, N_j^m(t_2) = 0) \\ &\quad = \prod_{k=1}^m P(N_i^k(t_1) = 0, N_j^k(t_2) = 0) \\ &\quad = \prod_{k=1}^m \sum_{l=0}^{+\infty} P(N_i^k(t_1) = 0 \mid N^k(t_2) = l) \\ &\qquad \times P(N_j^k(t_2) = 0 \mid N^k(t_2) = l)P(N^k(t_2) = l). \end{aligned}$$

Then, using Lemma 2.2, we obtain

$$\begin{aligned} &P(\tau_i > t_1, \tau_j > t_2) \\ &\quad = \prod_{k=1}^m \sum_{l=0}^{+\infty}\left(1 - \frac{\beta_i^k(t_1)}{t_2}\right)^l\left(1 - \frac{\beta_j^k(t_2)}{t_2}\right)^l \frac{e^{-\lambda_k t_2}(\lambda_k t_2)^l}{l!} \\ &\quad = e^{-\sum_{k=1}^m \lambda_k t_2\left[1-\left(1-\frac{\beta_i^k(t_1)}{t_2}\right)\left(1-\frac{\beta_j^k(t_2)}{t_2}\right)\right]}. \end{aligned}$$

The proof of (3) proceeds exactly as above.

This ends the proof. □

Remark 2.3. If we assume that $p_j^k(s) \equiv p_j^k$ for $k = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, then it follows from Eq. (3) that

$$P(\tau_1 > t_1, \tau_2 > t_2, \ldots, \tau_n > t_n) = e^{-\sum_{k=1}^m \lambda_k t_n\left(1-\prod_{j=1}^n\left(1-p_j^k\frac{t_j}{t_n}\right)\right)},$$

and

$$P(\tau_1 > t, \tau_2 > t, \ldots, \tau_n > t) = e^{-t\sum_{k=1}^m \lambda_k\left(1-\prod_{j=1}^n\left(1-p_j^k\right)\right)}. \tag{4}$$