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A coupled Markov chain approach to credit risk modeling ☆

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ABSTRACT

We propose a Markov chain model for credit rating changes. We do not use any distributional assumptions on the asset values of the rated companies but directly model the rating transitions process. The parameters of the model are estimated by a maximum likelihood approach using historical rating transitions and heuristic global optimization techniques.

We benchmark the model against a GLMM model in the context of bond portfolio risk management. The proposed model yields stronger dependencies and higher risks than the GLMM model. As a result, the risk optimal portfolios are more conservative than the decisions resulting from the benchmark model.

1. Introduction

In this paper, we present a coupled Markov chain (CMC) model which builds on the approach in Kaniovski and Pflug (2007). The aim of the model is to come up with a statistical description of the joint probabilities of credit rating changes of companies, which does not depend on distributional assumptions of the joint distribution of the asset values of the companies. We assume the that the individual rating transitions follow Markov processes and model the dependency between rating migrations of different companies by coupling the corresponding Markov chains. The advantage of being able to describe the dependencies of the credit quality of multiple debtors is that risk management on a portfolio level can be based on such a model. Therefore, we assess the quality of the model in the context of a stylized bond portfolio optimization problem and compare the portfolio decisions based on the proposed coupled Markov model with the decisions based on a GLMM model from the literature. The results show that the proposed model yields more conservative decisions then the GLMM model.

A major advantage of the proposed model over Kaniovski and Pflug (2007) is that it lends itself to statistical estimation of the parameters. More specifically, we derive the likelihood function of the model and develop methods for finding

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solutions to the maximum likelihood problem—a task which is complicated by the fact that the likelihood function is non-convex and computationally expensive to evaluate.

Although agency ratings have been criticized for their sluggish response to fast evolving events (see Altman, 1998; Crosbie and Bohn, 2002; Lando and Skødeberg, 2002; Nickell et al., 2000), many models use credit ratings as a basis for assessing credit risk. The credit rating of a company condenses a range of qualitative and quantitative assessments of the credit worthiness of a company and therefore is a signal for the credit quality of the debtor, which is consistent over time as well as among different debtors. Furthermore, rating based valuations are of increasing importance since pending new banking regulations use ratings as an important input for calculating capital requirements for banks (see Basel Committee on Banking and Supervision, 2004).

The most commonly used rating based method for modeling credit risk is the *CreditMetrics* approach. The main idea behind *CreditMetrics* is similar to the one in this paper: the current rating of a company influences the default probability in the next period. The difference, however, to the proposed approach is the copula used to specify the joint behavior of the rating processes for different companies. In the *CreditMetrics* approach, a Gaussian copula is used for this purpose. There exist a range of models describing the joint default behavior of companies in the literature: for excellent surveys and model classifications see for example Crouhy et al. (2000), Duffie and Singleton (2003), Frey and McNeil (2003), Frydman and Schuermann (2008), Gordy (2000) or McNeil et al. (2005).

In McNeil and Wendin (2006, 2007), the authors propose a generalized linear mixed model (GLMM) for rating transitions which is estimated using Bayesian techniques. The model describes systematic risk factors as a combination of fixed and random effects and also allows for serial correlations in the unobserved risk factors and hence for so called *rating momentum* on a macroeconomic scale.

Stefanescu et al. (2009) propose a model for continuous *credit worthiness* variables, which are translated to discrete ratings by identifying a rating class with an interval of the credit worthiness score. The continuous credit worthiness variables are allowed to depend on obligor specific as well as macroeconomic factors, whereby the latter are used for modeling dependencies in rating transitions of different obligors.

In Korolkiewicz and Elliott (2008) credit quality is modeled by a hidden Markov model. The published credit ratings are considered to be noisy signals that give an indication of the true credit worthiness.

Models for credit quality based on ratings are also frequently used in the pricing and risk management literature, see for example Jarrow et al. (1997), Kijima (1998), Kijima and Komoribayashi (1998).

Note that there is some empirical evidence hinting to the fact that the Markov assumption of credit ratings does not always hold (see Altman, 1998; Lando and Skødeberg, 2002; Nickell et al., 2000). The reasons for this might be contagion effects (cf. Giesecke and Weber, 2006) or long range dependencies in macroeconomic variables. Nevertheless, we do not consider more complicated models as the Markov assumption does not seem to be *too wrong* as shown in Kiefer and Larson (2007) and is implicit in most credit risk models.

This paper is structured as follows: Section 2 is devoted to a discussion of the coupled Markov chain model. In Section 3, we discuss a maximum likelihood approach which is subsequently used to estimate the parameters of the model from empirical data in Section 4. In Section 5, we compare the proposed model to a model by McNeil and Wendin (2006) and discuss the differences of two models in a risk management context. Section 6 concludes the paper.

2. The model

The model is based on the ideas presented in Kaniovski and Pflug (2007). For sake of clarity, we postpone the discussion of the differences to the aforementioned paper to the end of this section.

We model joint rating transitions of companies in different rating classes belonging to different industry sectors, such that

1. migrations of companies having the same credit rating are dependent;
2. evolution of companies through credit ratings are dependent;
3. every individual migration is governed by a Markovian matrix, which is the same for all the companies.

In line with Kaniovski and Pflug (2007), we assume that the rating migration process of each company is Markov with identical rating transition matrix and that these processes are coupled in such a way that they are statistically dependent because of their dependence on common systematic factors.

We start by considering a diversified portfolio consisting of debt obligations of different firms $n \in \{1, \dots, N\}$. The debtors are non-homogeneous in their credit ratings and belong to different industry sectors. Assume that there are M non-default rating classes. The ratings are numbered in a descending order so that 1 corresponds to the highest credit quality, while M is next to the default class. For example, in terms of the rating scheme of Standard and Poor's (S&P) we have, $1 \leftrightarrow AAA$, $2 \leftrightarrow AA$, $3 \leftrightarrow A$, $4 \leftrightarrow BBB$, $5 \leftrightarrow BB$, $6 \leftrightarrow B$, $7 \leftrightarrow CCC$, $8 \leftrightarrow CC$, $9 \leftrightarrow C$ and $10 \leftrightarrow D$ with $M=9$.

A company n is fully characterized by its rating x_n^t at time t as well as its industry sector $s(n)$. Denote by $S \in \mathbb{N}$ the number of different industry sectors in the model, i.e. $s(n) \in \{1, \dots, S\}$ for all $n \in \{1, \dots, N\}$. Note that the sectoral