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Response to updated mortality forecasts in life cycle saving and labor supply

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ABSTRACT

Historical evidence shows that demographic forecasts, including mortality forecasts, have often been grossly in error. One consequence of this is that forecasts are updated frequently. How should individuals or institutions react to updates, given that these are likewise expected to be uncertain? We discuss this problem in the context of a life cycle saving and labor supply problem, in which a cohort of workers decides how much to work and how much to save for mutual pensions. Mortality is stochastic and point forecasts are updated regularly. A Markovian approximation for the predictive distribution of mortality is derived. This renders the model computationally tractable, and allows us to compare a theoretically optimal rational expectations solution to a strategy in which the cohort merely updates the life cycle plan to match each updated mortality forecast. The implications of the analyses for overlapping generations modeling of pension systems are pointed out.

1. Introduction

Demographic forecasts, including mortality forecasts, have historically been more uncertain than has generally been believed (e.g., Alho, 1990), yet the media usually only discusses point forecasts. Even macroeconomic analyses that purport to address questions as to the sustainability of policies have often been carried out under a single population scenario.

Over the past two decades, various attempts have been made to include stochastic demographics both in generational accounting, which seeks to determine whether the current tax and entitlement rules will lead to the balancing out of public finances in the long run (e.g., Alho & Vanne, 2006), and in overlapping generations models (Samuelson, 1958), whose computational versions can provide realistic representations for national economies that include a pension system (cf. the articles referenced by Alho, Hougaard-Jensen & Lassila, 2008).

Overlapping generations models involve individual or household optimization with respect to savings and labor supply. The decisions are dynamic in nature, and depend on expectations about future demographics. A key issue is the exact way in which those expectations are assumed to be formed when mortality is stochastic.

Lassila and Valkonen (2008) assumed, for the sake of technical simplicity, that the decision maker knows future demographics without error. In such a set-up, the decision maker formulates a life cycle saving and labor supply plan for each sample path of mortality and never revises it.

More recently, Lassila, Valkonen, and Alho (2014) applied, apparently for the first time, what they call "the updated point forecast" approach to the overlapping generations computations. In this approach, the decision maker learns the most recent point forecast every period, assumes that it is without error, and optimizes accordingly. This is a major step forward, since the perfect foresight assumption is of course problematic in the context of stochastic demographics.

In the updated point forecast problem, the feasibility of the numerical calculations hinges on the assumption that

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the cohort behaves as if the updated forecast were without error. One can argue that this is probably the way in which most societies react to changing forecasts (cf. Alho, 2014). However, it is clear that this is not the theoretically optimal approach, since one can always do at least as well by taking the possibility of future updates into account in the optimization. The latter is the so-called rational expectations approach that was first proposed by Muth (1961). However, in general, the consideration of stochastic demographics with fully rational expectations leads to very complicated, if not intractable, computations (caused by the "curse of dimensionality").[1]

We will compare life cycle savings and labor supply decisions under different informational assumptions. In particular, we will compare decisions made under rational expectations about future mortality with those made using updated point forecasts. In the rational expectations problem, the decision maker is assumed to know exactly the conditional distribution of future mortality, given the current mortality. This information is provided to her in the form of a stochastic forecast. Since our model is Markovian, and since the state space is not overly large, we can overcome the computational problems related to rational expectations (cf. Filar & Vrieze, 1997).

The first question we aim to answer is the extent to which the updated point forecast problem can be interpreted as providing a good approximation of the rational expectations solution.[2] We will consider how much the consumer loses, in welfare terms, by considering only point forecasts instead of stochastic forecasts with rational expectations. In addition, we will be able to compare the optimal savings and labor supply decisions in the two cases.

A related question of independent interest is, should individual consumers be interested in demographic uncertainty? If the answer to this question is positive, the public should be provided with stochastic demographic forecasts instead of the usual point forecasts.

Alho and Määttänen (2008) considered a life cycle savings problem in which mortality was stochastic and followed a very simple Markovian model that was essentially a discrete approximation of the log-bilinear model of Lee and Carter (1992). In this paper, we provide a Markovian approximation of a more general mortality model that was estimated for Finnish females by Alho and Spencer (2005). We then expand the earlier economic setting to include not only savings, but also the supply of labor. The assumption will be that individuals enjoy both leisure and consumption, and that the two have to be balanced in order to maximize utility. The analysis is carried out for a cohort that has a funded pension system, or *tontine*, in which the cohort's savings accrue interest. Cohort survival is assumed to follow exactly the survival probabilities obtained from the Markovian approximation. Otherwise, the model includes the so-called aggregate uncertainty in mortality, but the idiosyncratic uncertainty that can be important in actual pension schemes is left out. Therefore, the model includes the most important elements used in the computation of overlapping generations models, such as that of Lassila, Palm, and Valkonen (1997).

We will start by discussing the cohort's decision problem in terms of a utility function, and give the full rational expectations formulation of the work/saving problem. Then, we provide the formulation of the decision problem that assumes updated forecasts in a Markovian setting. This is followed by a description of the general way in which one may obtain a Markovian approximation to a multidimensional predictive distribution. The results for expected life time utility are given in terms of welfare equivalents, and compared to those that have been obtained earlier in a simpler setting. We conclude by considering the implications of our findings for the application of computational overlapping generations calculations.

2. Decisions on work and savings and a pension system

2.1. Utility considerations

Consider a cohort of homogeneous individuals who enter working age at exact time $t = 0$ and live during $t = 0, 1, \ldots, T$. As of $s > 0$, the individuals are retired. During $t = 0, 1, \ldots, s-1$, the individuals have a time endowment of one unit that is split between work and leisure. In the beginning of each period t, the members of the cohort decide how much to work, $0 \leq w_t \leq 1$, and how much to consume, $c_t \geq 0$. The periodic utility is a function of consumption and time worked, $u_t = u(c_t, w_t)$.

We assume that leisure and consumption are both necessary for the creation of utility, and that the periodic utility function is of the form,

$$u(c, w) = ((c^\alpha(1-w)^\beta)^{1-\lambda} - 1)/(1-\lambda), \quad (1)$$

where $\alpha > 0, \beta > 0, \lambda > 0$ and $\lambda \neq 1$. For $\lambda = 1$, we have $u(c, w) = log(c^\alpha(1-w)^\beta)$. Following the definitions of Arrow (1971), this utility function is often considered to represent a constant relative risk aversion; that is, given the other parameters of the utility function, λ determines the degree of aversion to the relative risk.[3] The parameters α and β, in turn, determine the relative importance of consumption and leisure, respectively. Preferences are assumed to be time-separable, i.e., the utilities of different periods are additive.

Consumption is financed by wages that are proportional to the time worked. Therefore, there are no limitations to the assumption that the total wage in a period is equal to the time worked w. If there was no saving for the future, this would also be the consumption, or $c = w$. In this case, the utility can be re-expressed (with some abuse of notation) as $u = ((w^\alpha(1-w)^\beta)^{1-\lambda} - 1)/(1-\lambda)$. This is maximized at $w* = \alpha/(\alpha+\beta)$. However, during retirement ages, $w_t = 0$, so $u_t = u(c_t, 0)$, where the consumption must be financed from earlier savings.

1 See Hasanhodzic and Kotlikoff (2013) for a discussion of the curse of dimensionality in overlapping generations models.

2 This question relates our paper to the literature on boundedly rational decisions. For a recent study in the field, see for instance Winter, Schalfmann, and Rodepeter (2012).

3 Statisticians will notice that the functional form of the utility function is that of the so-called Box–Cox transformation (Box & Cox, 1964).