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Insider trading with public and shared information

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ABSTRACT

We study the impact of public information and shared information on traders' trading behavior in the context of Kyle's (1985) speculative market. We suppose that there are four types of traders in our model: one insider, *M* outsiders, liquidity traders, and market makers. We explicitly describe the unique linear Nash equilibrium and find that public information harms the insider but benefits the outsiders and noise traders. Also, the market is more efficient because of the existence of public information.

1. Introduction

Using an extension of the framework of Kyle (1985), this paper analyzes the impact of public information on the trading behavior of the insider and informed outsiders. As we know, when there is only one inside trader in the market, the value of information is always positive for the insider, and when more than one insiders have the same private information, the value of information is not always positive. "Competition" among inside traders has a big influence on the market structure. Kyle (1985) found that the monopolistic insider, in order to maximize his conditional profit, would trade in a recursive manner in discrete model, while in continuous time case as the time interval went to zero, the private information was incorporated into market price at a constant speed, and the market depth was constant over time (for the case of more than two insiders having the same private information see Holden and Subrahmanyam, 1992, which found that each trader tried to beat the others in the market and their information was revealed almost immediately).

Kyle (1985) has elicited a large body of literature. For example, Jain and Mirman (2000), Daher and Mirman (2007) and Wang et al. (2009), among others, explored various types of speculative markets by modeling the financial and real sectors together in order to study the insider's (or insiders') decision-making process and its effects on the output in the real sector, stock price of the firm in the financial sector, and the information revealed to the public. Luo (2001) extended Kyle's model (without the outsiders) by showing that when there exists public information, the monopoly insider put a negative weight on the public information in formulating his optimal strategy by moving the price in the direction of the asset value. Zhang (2008) analyzed dynamic market where outsiders shared part of the information about a security with a corporate insider and updated their incomplete information by learning from disclosed insider trades. Also, Rochet and Vila (1994), Huddart et al. (2001) and Decamps and Lovo (2006) have used variants of Kyle's model to analyze and explain real financial phenomena.

Inspired by Luo (2001) and Zhang (2008), we provide a model with public information and shared information with the presence of outsiders.[1] We will study the impact of public information and shared information on the behavior of the insider and informed outsiders in a speculative market with four kinds of traders: one risk neutral insider, *M* risk neutral outsiders, noise traders, and competitive risk neutral market makers. The main finding is that public information and shared information are harmful for the insider but beneficial to noise traders and informed outsiders; when the size of outsiders *M* is large enough, a larger *M* means a higher expected profit for the insider; all outsiders prefer smaller *M*; and noise traders prefer larger *M*.

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[1] The idea is a combination of Luo (2001) and Zhang (2008), but the information structure in our model is different from their models.

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This paper is structured as follows. In Section 2, we present the model. In Section 3, we identify the unique linear Nash equilibrium of the model. Then in Section 4 analyze the properties of the equilibrium. In Appendix A we provide proofs.

2. The model

Consider a model with four kinds of traders: one risk neutral insider (the informed trader), M ($M \geq 2$) risk neutral outsiders, noise traders, and competitive risk neutral market makers. There are two periods (period 0 and period 1) and a single risky asset in the economy. At period 0, the public and private information is released and the trading takes place; and at period 1, the risky asset payoff is realized. The ex-post liquidation value of the risky asset is a random variable $\tilde{v} = \tilde{\zeta} + \tilde{s} + \tilde{\epsilon}$, which is normally distributed with mean p_0 and variance σ_v^2 i.e.

$$\tilde{v} \sim N\left(p_0, \sigma_v^2\right).$$

The first component, $\tilde{\zeta}$, is related to the private information known only to the insider. Prior to trading, the insider learns the value of the security by observing the signal $\tilde{\zeta}$, $\tilde{s}$ and $\tilde{\epsilon}$. The second component, $\tilde{s}$, is related to the shared information obtainable by every outsider (but not by market makers and noise traders). Every outsider can observe $\tilde{s}$ at time 0, which is drawn from an independent and normal distribution with zero mean and variance $t_s\sigma_v^2$. And the third part, $\tilde{\epsilon}$, which is normally distributed with zero mean and variance $t_\epsilon\sigma_v^2$, is related to the information known to all traders in the market (including market makers but no noise trader).[2] We call $\tilde{\epsilon}$ public information. Thus the insider's information (public and private) is ($\tilde{v}$, $\tilde{s}$, and $\tilde{\epsilon}$), and every outsider's information is ($\tilde{s}$, $\tilde{\epsilon}$). Noise traders have an inelastic demand for the risky asset and their trading is exogenous. The quantity traded by noise traders, denoted by $\tilde{u}$, is normally distributed with mean zero and variance σ_u^2. The random variables $\tilde{\zeta}$, $\tilde{u}$, $\tilde{\epsilon}$ and $\tilde{s}$ are mutually independent. The presence of the noise traders not only serves as a camouflage and makes it impossible for the market makers to exactly infer the insider's and the outsiders' private information, but also is the source of the profits to be exploited by the insider and informed outsiders.

We conform the trading process to Luo (2001). The trading proceeds as follows. At period 0, the public and private information is announced. After receiving the information the insider chooses his trading strategy by submitting an order $\tilde{x} = X(\tilde{v}, \tilde{s}, \tilde{\epsilon})$ and each outsider $i \in \{1, \ldots, M\}$ chooses his trading strategy by submitting an order $\tilde{y}_i = Y_i(\tilde{s}, \tilde{\epsilon})$. Also, the trading volume of noise traders, $\tilde{u}$, is realized. The insider and every outsider choose the quantity they trade based on their information. When doing so, they can observe their individual information but do not know $\tilde{u}$ and the others' quantities traded. The market makers observe the total order $\tilde{y} = \tilde{x} + \tilde{y}_1 + \tilde{y}_2 + \cdots + \tilde{y}_M + \tilde{u}$, but not each individual's. After receiving the total order $\tilde{y}$, the market makers take the opposite side of the incoming order and set the price $\tilde{p} = P(\tilde{\epsilon}, \tilde{y})$ of the risky asset in a semi-strong efficient way such that they expect to earn a zero profit conditional on the public information and the total order. At period 1, the uncertainty is resolved and the risky asset payoff is realized. The structure of the economy is common knowledge.

The equilibrium conditions are that the competition between market makers drives their expected profits to zero conditional on the order flow and that the insider and every informed outsider choose their trading strategy to maximize their expected profits. Following the convention in the existing literature, an equilibrium is said to be linear if the pricing rule is an affine function of the order flow. We will give the definition of equilibrium in the following section.

[2] We assume t_s, $t_\epsilon > 0$ and $t_s + t_\epsilon < 1$. The notations of variances used here are for latter computational convenience.

3. The unique linear equilibrium

In this section we give the equilibrium concept of the model prescribed in the preceding section, and state the existence of a unique linear equilibrium.

A strategy for the informed insider is given by a measurable function $X: R^3 \to R$, which determines his market order as a function of his observable information. For a given strategy X, the insider's corresponding demand of the asset will be $\tilde{x} = X(\tilde{v}, \tilde{s}, \tilde{\epsilon})$.

For an outsider $i \in \{1, \ldots, M\}$, his strategy is given by a measurable function $Y_i: R^2 \to R$, which specifies his market order as a function of his available information. For a given strategy Y_i, let $\tilde{y}_i = Y_i(\tilde{s}, \tilde{\epsilon})$. A strategy combination $(X, Y_1, Y_2, \cdots, Y_n)$ determines the order flow as

$$\tilde{y} = \tilde{x} + \tilde{y}_1 + \tilde{y}_2 + \cdots + \tilde{y}_M + \tilde{u}.$$

The market makers observe the public information and the realization of the order flow, but not any of its components, and engage in a competitive auction to serve the order flow. The outcome of this competition is described by a measurable function $P: R^2 \to R$ which specifies the pricing rule that brings them zero expected profit. Given $(P, X, Y_1, Y_2, \cdots, Y_n)$, denote $\tilde{p} = P(\tilde{\epsilon}, \tilde{y})$ and let $\tilde{\pi}(X, P) = (\tilde{v} - \tilde{p})\tilde{x}$, $\tilde{\pi}_i(Y_i, P) = (\tilde{v} - \tilde{p})\tilde{y}_i$ denote the resulting trading profit of insider and that of the ith outsider, respectively.

Definition 1. $(P, X, Y_1, Y_2, \cdots, Y_M)$ is an equilibrium for the one-shot model if

(1) Profit maximization: for any alternate trading strategy X' of the insider,

$$E[\tilde{\pi}(X, P) \mid \tilde{v}, \tilde{s}, \tilde{\epsilon}] \geq E[\tilde{\pi}(X', P) \mid \tilde{v}, \tilde{s}, \tilde{\epsilon}].$$

For any alternate trading strategy Y_i' of outsider i,

$$E[\tilde{\pi}_i(Y_i, P) \mid \tilde{s}, \tilde{\epsilon}] \geq E[\tilde{\pi}_i(Y_i', P) \mid \tilde{s}, \tilde{\epsilon}],$$

where $i = 1, 2, \cdots, M$.

(2) Market efficiency: $P(\tilde{\epsilon}, \tilde{y}) = E(\tilde{v} \mid \tilde{\epsilon}, \tilde{y})$.

Remark 3.1. If we view the competitive market makers as a single player, the above notion of equilibrium is the concept of Nash equilibrium. The game is then among the insider, M outsiders, and the representative market maker. The latter's objective can be prescribed as maximizing $-E\left[(\tilde{v} - \tilde{p})^2\right]$ subject to the constraint that $\tilde{p}$ is measurable with respect to the sigma field generated by $\tilde{\epsilon}$ and $\tilde{y}$. It is well known that the solution of this problem is the conditional expectation $P(\tilde{\epsilon}, \tilde{y}) = E(\tilde{v} \mid \tilde{\epsilon}, \tilde{y})$.

Now we state a well known regression result that will be used later.

Lemma 3.1. Let X_1 and X_2 have joint normal distribution, $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N(\mu, \Sigma)$ with $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Then the random variable X_1 conditional on X_2 has a normal distribution, and

$$E[X_1 \mid X_2] = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2), Var(X_1 \mid X_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}.$$

Definition 2. We say an equilibrium is a linear equilibrium if the strategy functions X, Y_i, $i = 1, \ldots, M$ and P are all affine functions. That is, there exist constants a, b_i, c, α, β, θ, γ_i, δ_i, η and λ such that

$$\tilde{x} = X(\tilde{v}, \tilde{s}, \tilde{\epsilon}) = a + \alpha\tilde{\epsilon} + \beta(\tilde{v} - p_0) + \theta\tilde{s}, \tag{3.1}$$

$$\tilde{y}_i = Y_i(\tilde{s}, \tilde{\epsilon}) = b_i + \gamma_i\tilde{s} + \delta_i\tilde{\epsilon}, i = 1, \cdots, M, \tag{3.2}$$

$$\tilde{p} = P(\tilde{\epsilon}, \tilde{y}) = c + \eta\tilde{\epsilon} + \lambda\tilde{y}. \tag{3.3}$$

Now we state our main result of this paper.