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Price functionals with bid–ask spreads: an axiomatic approach

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Abstract

In Jouini and Kallal [Jouini, E., Kallal, H., 1995. Martinagles and arbitrage in securities markets with transaction costs. Journal of Economic Theory 66 (1) 178-197], the authors characterized the absence of arbitrage opportunities for contingent claims with cash delivery in the presence of bid–ask spreads. Other authors obtained similar results for a more general definition of the contingent claims but assuming some specific price processes and transaction costs rather than bid–ask spreads in general (see for instance, Cvitanic and Karatzas [Cvitanic, J., Karatzas, I., 1996. Hedging and portfolio optimization under transaction costs: a martinangle approach. Mathematical Finance 6, 133-166]). The main difference consists of the fact that the bid–ask ratio is constant in this last reference. This assumption does not permit to encompass situations where the prices are determined by the buying and selling limit orders or by a (resp. competitive) specialist (resp. market-makers). We derive in this paper some implications from the no-arbitrage assumption on the price functionals that generalizes all the previous results in a very general setting. Indeed, under some minimal assumptions on the price functional, we prove that the prices of the contingent claims are necessarily in some minimal interval. This result opens the way to many empirical analyses.

Keywords: Price functionals; Bid–ask spreads; No-arbitrage assumption; Market-makers; Contingent claims

1. Introduction

There an important literature on the contingent claims pricing problem under transaction costs on the primitive assets. For instance, Leland (1985) studied the replication price for a contingent claim in a discrete time setting. In this paper, when the horizon is kept fixed and

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the number N of time periods goes to infinity, the price of the primitive asset is assumed to converge to a diffusion process. If we further assume that the transaction costs go to zero as the square root of N, Leland (1985) claims then that the replication price for a call option converges to the Black and Scholes price of this option in a model without transaction costs but with a correctly modified volatility for the primitive asset. For a correct proof of Leland's result see Kabanov (1997). In Boyle and Vorst (1992), the authors do not assume that the transaction costs go to zero and characterize the replication cost as an integral of the future prices relatively to a signed measure which is not, in general, a probability measure as in the frictionless model.

Bensaid et al. (1992) in the same year revolutionized the transaction costs literature considering dominating strategies instead of replicating ones. Indeed, the authors note that the replication cost is not necessarily, as in the transaction costless framework, the minimum cost necessary to obtain at least the same payoffs as those of the considered contingent claim. They propose then, in a discrete time setting, an algorithm in order to compute the so-called domination price: the minimum cost necessary to obtain at least the same payoffs as those of the considered contingent claim. Furthermore, they characterize the situation where the replication price is equal to the domination price and where the replication strategy is in some sense optimal.

In the same year and after the seminal work of Bensaid et al. (1992), Jouini and Kallal characterized, in a paper published in 1995, this domination price in a general setting. They prove that this price is equal for a given contingent claim to the supremum of the future payoffs expected value. This supremum is taken over all the equivalent martingale measures associated to one of the processes lying between the bid and the ask price processes. Furthermore, they characterize the absence of arbitrage opportunities in the model by the existence of a process lying between the bid and the ask price processes and of an equivalent probability measure for which the considered process is a martingale.

More recently, Shirakawa and Konno (1995) in a stationary binomial framework, Kusuoka (1995) in a discrete time and finite number of states of the world framework and Cvitanic and Karatzas (1996) in a diffusion setting, obtained results similar to some of Jouini and Kallal (1995a) [2] in a different setting. Indeed, in Jouini and Kallal (1995a), the authors only consider contingent claims with cash delivery. Note that this restriction is innocuous in the transaction costless framework but this is not at all the case in our framework.

Nevertheless, it is important to remark that in all these papers, the authors assume the existence of some price process S satisfying some classical conditions implying the absence of arbitrage opportunities in a frictionless framework (diffusion, binomial process. . .). The bid and the ask price processes are obtained multiplying S by $(1+\lambda)$ and $(1-\mu)$. In this setting, the transaction costs are proportional to the price S and the bid and ask price processes have the same behaviour. The Jouini and Kallal (1995a) paper is the only one with two independent price processes: a bid price process and an ask price process. The bid–ask spread can be interpreted as transaction costs but can be explained by the buying and selling limit orders on the markets. These prices are the prices for which a buyer or a seller is sure to find an immediate counterpart. From this point of view the bid–ask spread includes

[2] For instance, Cvitanic and Karatzas do not characterize the absence of arbitrage opportunities but only the domination price. Indeed, the choice of a diffusion framework implies the absence of arbitrage opportunities.