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On second-price auctions and imperfect competition

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Abstract

Consider two sellers each of whom has one unit of an indivisible good and two buyers each of whom is interested in buying one unit. The sellers simultaneously set reserve prices and use second-price auctions as rationing device. An equilibrium in pure strategies where each seller has a regular customer is characterized. The result is applied in order to demonstrate that *not* allowing sellers to use second-price auctions may enhance total surplus.

JEL classification: D43; D44; D82

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1. Introduction

While there exists a vast literature on auctions and optimal selling mechanisms for the case of privately informed buyers,[1] most papers in this literature are focused on a monopolistic seller. In contrast, in this paper I consider two capacity constrained sellers who compete by setting prices and who may use second price auctions as rationing devices if more buyers show up than there are goods. I analyze this situation in the simplest possible model where the buyers' valuations are independently distributed.

The virtues of a second-price auction in the monopolistic framework are by now well understood. In particular, in accordance with the textbook analysis of monopolistic price

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[1] For surveys, see, e.g. McAfee and McMillan (1987), Matthews (1995), or the recent book by Krishna (2002).

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discrimination, the expected total welfare would clearly be reduced if a monopolist were not allowed to use a second-price auction and could instead only set a fixed price (and select the winning buyers randomly if there are more buyers than goods).

At first glance, one might guess that allowing the use of second-price auctions as rationing device must also be welfare enhancing in the case of competing sellers. After all, with an auction a seller can extract more additional surplus when he attracts more buyers by decreasing the price, so that we might expect lower prices and thus more trade. However, the willingness of a buyer to go to a cheaper seller can be smaller if the buyer knows that the seller will extract more of the surplus by using an auction. Thus, a price reduction can be less profitable, so that in fact higher prices could be sustainable if sellers are allowed to use second-price auctions. As a consequence, second-price auctions might be anti-competitive, so that total welfare could be increased if sellers were not allowed to price discriminate.

I am aware of only a few other papers that analyze auctions in a competitive framework, in particular McAfee (1993), Peters and Severinov (1997) and Burguet and Sákovicz (1999).[2] These papers are mainly focused on the case of large markets with many buyers and/or sellers, where buyers randomize among sellers. In contrast, I analyze a duopolistic situation and focus on pure strategies. This paper is also related to Wang (1993), who compares auctions and posted prices in the monopolistic framework (his results are driven by the assumption that auctioning is costly, which is not made here).

2. The model

There are two risk-neutral sellers, A and B, each of whom possesses one unit of an indivisible good. For simplicity, the sellers' valuations are assumed to be zero. Moreover, there are two potential buyers, $i \in \{1, 2\}$. Buyer i's willingness to pay for one unit of the good is denoted by $v_i \in [0, 1]$. The buyers' valuations are private information; yet, it is commonly known that they are independently and symmetrically distributed according to the distribution function F. The density function is denoted by f. For simplicity, let us assume that F is sufficiently well behaved, such that the first-order approach will be applicable.[3] In particular, the well-known monotone hazard rate property $(\mathrm{d}((1 - F(v))/f(v))/\mathrm{d}v) < 0$ is supposed to hold; i.e. we are in Myerson's (1981) 'regular case'. Following the mechanism design approach, it is assumed that the sellers can make take-it-or-leave-it offers to the buyers. Specifically, a seller can commit not to sell her good if no buyer is willing to pay the price that she has posted.

Consider the following game. At date 1, the sellers A and B simultaneously announce reserve prices p_{A} and p_{B}, respectively. At date 2, the buyers simultaneously decide whether to go to seller A, to seller B, or to no seller at all. If only one buyer shows up at seller j's store, he receives the good and pays the price p_j, $j \in \{\mathrm{A}, \mathrm{B}\}$. If two buyers show up, seller j determines the winner of the good by a second-price auction with p_j as minimum bid.

[2] See also the related work on common agency and the revelation principle (Page and Monteiro (2001), Martimort and Stole (2002)) and on decentralized trading mechanisms (Coles (1998), De Fraja and Sákovics (2001), and the literature discussed there).

[3] Specifically, it is assumed that the maximizer of expression (2) is characterized by the first-order condition. It can be checked that this is, e.g. satisfied in the case of the uniform distribution.