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Harmonic symmetries of imperfect competition on circular city

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ABSTRACT

Taking location as given, we study imperfect competition on a circular city. In Bertrand oligopoly, we identify price harmonics as a function of firm unit costs and locations. The sum of oligopoly profits is larger when costs and/or locations are more dispersed in the 'dihedral majorization' sense. This also tends to be the case in which prices are more variable. We study how phase shifts between cost parameters and inter-firm distance parameters change production and oligopoly profits. An exact characterization of production patterns is developed in terms of the eigenvalues for what we call the price harmonics matrix. The same techniques are applied to Cournot oligopoly with spatial externalities on circular city. Solutions are compared with first-best. Production patterns can differ markedly when cost spillovers are negative.

1. Introduction

Much of economic analysis is concerned with external effects. These effects can be very diffuse to the extent that impacts on affected parties do not depend on location, as in a homogeneous good oligopoly model. Or there can be differential impacts, where the extent of direct impacts could be ordered by proximity, as with ingestion of second-hand smoke, damage from acid rain, and damage to product reputation due to publicized failure of a similar product. The general issue of modeling proximity is important because many, if not most, externalities impact other agents in a manner that depends on how similar an agent is to the externality originator. Our intent in this paper is to provide an approach to modeling externalities when distance between agents influences the extent of the externality. We will model firms competing for customers on circular city where the strategic behavior of a firm most directly affects adjacent firms.

Circular city was chosen because, as recognized by Salop (1979) and many others, model results are not driven by topological discontinuities that would obscure economic insights. Salop's study of price competition on circular city stimulated further inquiries into strategizing when differentiation arises due to location on a seemless loop, such as Eaton and Holtz Wooders (1985). These articles held location to be exogenous. Gupta et al. (2004) and others have sought to rationalize equilibrium location in a locate-then-compete game. Concurrent with the growing body of work on the theory of spatial interactions, developments in empirical methods are allowing for a clearer understanding of when and how economic distance matters (Anselin, 1988; Conley, 1999).

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Taking location as exogenous, we seek to characterize the roles of location and cost in strategic choices. Our model comes in two variants, both involving firms with constant marginal costs competing on circular city. The first is a shopping model, involving Bertrand pricing where non-discriminatory pricing strategies place arbitrage constraints on adjacent firms. The second involves the incidence of a production externality on an adjacent firm when firms compete over quantity strategies to produce an homogenous good.

The paper commences with a brief explanation of some relevant tools. As with Dixit (1986), Salant and Shaffer (1999), and others, we will exploit symmetries that arise naturally in oligopoly models. Group symmetries have provided insights on demand theory, as in Russell and Farris (1993), and also on economic index number theory (Barnett and Choi, 2008). The discrete group symmetry tools we use have also been employed by Pęski (2007) when modeling patterns as learned information upon which to make decisions. Hennessy and Lapan (2003, 2005) modeled portfolio choice and preferences over prices for a price-taking agent using discrete groups. In our case, though, the natural symmetries arising from local competition allow for a more developed analysis. We can work with the cyclic and dihedral symmetries, using harmonic representations that best capture spatial neighborhood effects. Ioannides (2006) takes this approach when studying social interaction. More closely related to our work, Picard and Tabuchi (in press) use harmonic methods to characterize how transportation costs affect equilibrium spatial structure.

For Bertrand competition, we establish equilibrium and use harmonic analysis to study some of the symmetries in equilibrium. In particular, a characterization of cost/separation parameter dispersions (i.e. a majorization relation) that is tailored to the symmetries of how firms are located is shown to increase price variance and aggregate oligopoly profit. A stronger alignment of cost and separation parameters in the phase-shift sense has the same effect. We show that price variance is always smaller than variance in exogenous parameters. This is due to price smoothing arising from competition at the margin. The difference depends on the harmonics of exogenous parameters; firm costs and inter-firm distances in our model. High frequency harmonics on exogenous parameters around the circle have less influence on price variance than do low frequency harmonics. This is because prices are strategic complements, and high heterogeneity versus adjacent firms impedes the capacity of less competitive firms to price high while increasing the latitude for more competitive firms to do so.

Nonetheless, we show that prices and exogenous parameters always exhibit positive covariance. How sales and exogenous parameters covary depends on dispersion in locations, dispersion in costs, and how costs covary with inter-firm distances. The spatial pattern of firm sales has a more involved relationship with exogenous parameters than does the pattern of firm prices. Firms price high when costs are high and competitors are remote. Firm sales are low when costs are high and high when competitors are remote. When cost and remoteness characteristics correlate positively across firms, then prices will tend to correlate positively with both but firm sales may express little if any pattern. A comparison with first-best shows that firms with comparatively remote competitors serve a customer base that is too small.

The quantity-setting variant of our framework is somewhat different in modeling spatial spillovers. Spillovers are chosen to be one-sided in direction along the circle and they can be strategic complements or substitutes, whereas in Bertrand competition they must be two-sided and must be strategic complements. Consequently the solution characteristics differ, having less symmetry. Surprisingly in one regard, concerning the effects of dispersion in cost parameters on oligopoly profits, the less structured symmetries of our Cournot model do not weaken our results. In a linear model we establish that efficient outputs correlate negatively with Cournot outputs when spatial cost spillovers are negative. A brief discussion concludes.

2. Preliminaries

Write $\Omega_N = \{0,1,\ldots,N-1\}$. With superscript T as the transpose operation, vector $u = (u_0,u_1,\ldots u_{N-1})^{\mathrm{T}}$ may be rotated as follows. Specify the $j+k+1$th ordinate, modulus N, as $u_{[j+k]}$ so that values $N=6$, $j=3$, and $k=5$ provide $u_{[j+k]} = u_2$ while $u_{[j-4]} = u_5$. We intend to compare the welfare implications of different cost and demand parameter vectors. To this end, some definitions are required.

Definition 1. Vector $u' \in \mathbb{R}^N$ is *cyclic majorized* by $u'' \in \mathbb{R}^N$ (written as $u' \leq_{\mathrm{cyc}} u''$) if there exist simplex weights $(\lambda_0, \lambda_1,\ldots,\lambda_{N-1})$, $\lambda_j \geq 0 \forall j \in \Omega_N$, $\sum_{j \in \Omega_N} \lambda_j = 1$, such that[1]

$$u' = \sum_{j \in \Omega_N} \lambda_j L^j(u''); \quad L^j(u'') = (u''_j, u''_{[j+1]}, u''_{[j+2]}, \ldots, u''_{[j-1]})^T. \tag{2.1}$$

Definition 2. Vector $u' \in \mathbb{R}^N$ is *dihedral majorized* by $u'' \in \mathbb{R}^N$ (written as $u' \leq_{\mathrm{dih}} u''$) if there exist simplex weights $(\lambda_0, \lambda_1,\ldots,\lambda_{2N-1})$, $\lambda_j \geq 0\ \forall j \in \Omega_{2N}$, $\sum_{j \in \Omega_{2N}} \lambda_j = 1$, such that[2]

[1] See Giovagnoli and Wynn (1996) on cyclic majorization.

[2] If $\lambda_j = 0\ \forall j \in \{N, N+1,\ldots,2N-1\}$, then $\leq_{\mathrm{dih}}$ reduces to $\leq_{\mathrm{cyc}}$. Dihedral majorization is more discerning in that $\lambda_j \neq 0\ \forall j \in \{N, N+1,\ldots,2N-1\}$ allows for comparison between vectors not preordered under cyclic majorization. Marshall and Olkin (1979) and Niezgoda (1999) provide references to work on the more general concept of group majorization. As far as we know this paper is the first to study dihedral majorization specifically.