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A maximum (non-extensive) entropy approach to equity options bid–ask spread

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HIGHLIGHTS

- We apply the Kaniadakis entropy to model options bid–ask spreads.
- The options spreads depend on distribution tails and the implied volatility.
- Information exclusively flows from the underlying asset to its derivatives.

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ABSTRACT

The cross-section of options bid–ask spreads with their strikes are modelled by maximising the Kaniadakis entropy. A theoretical model results with the bid–ask spread depending explicitly on the implied volatility; the probability of expiring at-the-money and an asymmetric information parameter (κ). Considering AIG as a test case for the period between January 2006 and October 2008, we find that information flows uniquely from the trading activity in the underlying asset to its derivatives. Suggesting that κ is possibly an option implied measure of the current state of trading liquidity in the underlying asset.

1. Introduction

Modelling options bid–ask spreads (or, options spreads), while considering the entire cross-section of strikes and (or) maturities, is challenging. This paper provides an approach to this challenge that is both empirical and theoretical. In essence, it relies on maximising the [1] entropy. Specifically, it assumes a competitive market-maker that maximises the Kaniadakis entropy, subject to his own valuation of calls and puts true values. This behaviour is due to suspected presence of informed traders, either in the derivative or underlying markets.

When information is asymmetric the definition of implied densities of future states is uncertain. A market-maker may then adjust the bid and ask quotes, accordingly and relative to subjective valuation of assets true values. This is done by a power-law transformation of the asset implied density. This density is defined by a rationale embedded in Ref. [2] maximisation of entropy in non-extensive systems (see also: Refs. [3–5]). Hence, this distribution estimate, generalises Boltzmann's density that subsumes systems are complete (namely, fully countable and accountable). The power parameter transforming this density is the κ parameter, that includes Boltzmann [6] entropy as a special case.

This paper provides a pricing model that assumes asymmetric information. It is justified empirically and theoretically by the inherent asymmetry in option spreads [7]. In this paper the bid–ask spread is written as an expected payoff, weighted by the generalised (Kaniadakis) κ-logarithm. Then, options spreads are shown to be related to their: implied volatility; probability of expiring at-the-money; premium and κ-parameter. Moreover, for a call, the bid–ask spread is shown to decrease with respect to the strike price.

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An empirical study of traded options on AIG equity between 2006 and 2008 is conducted. We find (by means of Ref. [8] causality and the Transfer entropy) that overall, information flow almost uniquely from the relative bid–ask spread of the underlying asset to the option implied parameter-κ. Our result confirms the predictions made by Ref. [9] derivative hedge theory. This result is verified for the pre- and post-crisis periods, using AIG data.

Finally, our analysis precluded the problem of bid–ask asymmetry (i.e.: the location of the true option value with respect to its bid and ask quotes). The latter not to be neglected, since true options values are usually close to their respective bid quotes [10]. In such cases, the implied volatility smile is less pronounced compared to that derived from options mid-quotes. Although measuring this asymmetry is not a essential aim of this paper, we do provide an empirical methodology to measure and parametrise it.

The approach we use intersects two aspects of the vast literature on options and their informational contents. A first aspect is concerned with the inference of option implied risk-neutral densities and their informational contents. Specifically, our methodology confirms the entropy based approach to estimating option implied densities (for example: Refs. [11–14]). An advantage in using the maximum entropy framework is its lack of assumptions regarding the distributional laws governing the underlying asset price dynamics. Moreover, it maintains the no-arbitrage condition and obeys the martingale restriction [15]. This paper extends this framework by maximising the [1] entropy, which yields an option implied density that exhibits the power-law property. Note that a power-law density can also be obtained by maximising the [2] entropy. However, the [2] entropy measures an “expected distance” between *complete* and *incomplete* distributions. While the Kaniadakis entropy, measures the “expected distance” between an *incomplete* and an *over-complete* distribution. It further accentuates, therefore, the degree of uncertainty regarding statistical information in a state of incompleteness.

A second aspect of this paper is both theoretical and empirical providing a theoretical explanations of options spreads. Theoretically, there are two opposite explanations to observed options spreads: the asymmetric-information or price-discovery in the option market [16] and the derivative hedge theory [9]. The former predicts that options spreads are due to active informed traders trading in the option market, implying informed traders prefer the option market. They exploit market incompleteness, for which a bidirectional relationship between options and their underlying asset is an evidence of their mutual effects. [9] on the other hand, explain that options spreads are directly related to liquidity risk in the underlying market. They point out that the market maker is unable to perfectly hedge his positions. Therefore, options spreads compensate the market-maker for the liquidity risk exposure. Hence, they are positively related to informed trading in the underlying asset. This provides an empirical support for these contributions, elaborating the effects of information asymmetry to a system non-extensiveness.

This paper is divided into four sections. The first section briefly presents the Kaniadakis entropy. The second section describes our approach to the modelling of the bid–ask spread. The third section presents the empirical results and the last section concludes this work.

2. The Kaniadakis entropy

The [1] entropy is a one parameter entropy, generalising Boltzmann [6] entropy and is a special case of the [17] entropy (see also: Refs. [18,19]). The functional form of this entropy is indicated in Eq. (1).

$$H_\kappa\left[f(x)\right] = -\int f(x)\ln_{\{\kappa\}} f(x)\mathrm{d}x \tag{1}$$

With the following properties:

- $\ln_{\{\kappa\}}(x) = \frac{x^\kappa - x^{-\kappa}}{2\kappa}$ is the κ-logarithm.
- $\exp_{\{\kappa\}}(x) = \exp(\frac{1}{\kappa}\arcsin h(\kappa x))$, such that $\exp_{\{\kappa\}}(\ln_{\{\kappa\}} x) = x$.
- For $\kappa \to 0$, $\ln_{\{\kappa\}}(x) \to \ln(x)$ and $\exp_{\{\kappa\}} x \to \exp(x)$.

Further, letting $\ln_{\{\kappa\}}(x)$ be written in terms of two [2] logarithms we have the Kaniadakis entropy in Eq. (2):

$$H_\kappa\left[f(x)\right] = -\frac{1}{2\kappa}\int\left[\frac{1}{1+\kappa}f^{1+\kappa}(x) - \frac{1}{1-\kappa}f^{1-\kappa}(x)\right]\mathrm{d}x. \tag{2}$$

Optimising the expression in Eq. (2) (subject to some moments constraints) enables to easily recover an explicit mathematical expression for the probability density function $f(x)$, which is a power-law.

Given a set of N moment constraint, $E(g_i(x)) = \int g_i(x)f(x)\mathrm{d}x$, a probability distribution function is inferred by maximising the expression in Eq. (2). In other words:

$$\begin{aligned}&\underset{f(x)}{\text{Max}}\quad H_\kappa\left[f(x)\right] = -\frac{1}{2\kappa}\int\left[\frac{1}{1+\kappa}f^{1+\kappa}(x) - \frac{1}{1-\kappa}f^{1-\kappa}(x)\right]\mathrm{d}x\\ &\text{subject to } E(g_i(x)) = \int g_i(x)f(x)\mathrm{d}x, \quad i = 1,\dots,N.\\ &\text{and}\quad \int f(x)\mathrm{d}x = 1.\end{aligned} \tag{3}$$