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An imperfect competition on scale-free networks

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HIGHLIGHTS

- We build imperfect competitions on networks of rivalry relationships among firms.
- Output and price are always the same as those in the monopoly on scale-free networks.
- The above result has sharp contrast to ordinary imperfect competition.

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ABSTRACT

We study an imperfect quantity competition on networks that represent rivalry relationships among firms. We show that the more heterogeneous the underlying network is, the more the output and the price are. The output and the price on scale-free networks are counter-intuitively the same as those in the monopoly regardless of the number of rival firms. We also show that any inverse demand function represented by a network has the corresponding utility function, which justifies the inverse demand function.

1. Introduction

In the present paper we consider for the first time the dynamics of an economic model, namely imperfect quantity competition on complex networks. We show that the behavior of the model is significantly different on scale-free networks from the standard quantity competition in the economics literature.

Complex networks, and particularly scale-free networks, have attracted much attention in the past decade [1–11]. It was believed for a long time that most real-world networks were random networks. Many real-world networks, however, have been found more heterogeneous than the random ones [12], having a lot of vertices with few links and a few vertices with lots of links. Some of them are scale-free networks with the degree distribution $\Pr(k) \sim k^{-\gamma}$, where k denotes the degree (the number of the links attached to a vertex) and the exponent γ is a real number. Almost all scale-free networks in reality have the exponent γ in the range $2 < \gamma \leq 3$. We hence focus on such networks in the present paper, calling them typical scale-free networks. Of course, not all real-world networks follow the scale-free distribution precisely in the whole range of their degree distributions, but scale-free networks have been studied greatly because the tail part of the degree distribution often follows the scale-free one, which matters most. Indeed, the mean degree of the *nearest neighbors* $\langle k_{\text{nn}} \rangle$, which plays an essential role in the present paper, is determined mostly by the tail part of the degree distribution. The more heterogeneous a network is, the greater the mean degree of the nearest neighbors $\langle k_{\text{nn}} \rangle$ is.

Dynamics of models residing on complex networks is also of great interest. Their behavior strongly depends on the underlying network structure. This point was already known prior to the advent of scale-free networks. Unlike random and homogeneous ones, however, a scale-free network has great heterogeneity, which affects the behavior of the residing model drastically. We will indeed demonstrate in the present paper that the network heterogeneity increases the output and the

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price in imperfect competition. In particular, we will show that the output and the price on scale-free networks are always equal to those of the monopoly solution irrespective of the number of rival firms.

The following papers demonstrated that the underlying network, particularly a scale-free one, changes the behavior of the residing model. Refs. [13–18] showed how the underlying network structure significantly affects the emergence of cooperation in evolutionary games [19] of the prisoners' dilemma. Refs. [20,21] showed the same for games of public goods provision on networks. Ref. [22] showed that an epidemic explosion always happened on scale-free networks with the exponent $\gamma \leq 3$. Ref. [23] studied cooperation on networks in a laboratory experiment. Economic phenomena on networks have been studied in the following papers, although scale-free networks were not featured. Refs. [24,25] discussed employment and financial contagion on networks, respectively. Ref. [26] studied a model of network formation. Ref. [27] examined the efficiency and stability of social and economic networks.

The outline of the present paper is as follows. In Section 2, imperfect competition on networks will be introduced. The networks represent rivalry relationships among firms. Further, we will introduce a class of utility functions from which such a network relationship is derived in Appendix A. We will introduce a mean-field approximation for imperfect competition, first on homogeneous networks in Section 3 and then on heterogeneous networks and scale-free networks in Section 4, which is the main part of the paper. In Section 5, we will confirm the mean-field approximate solutions by numerical simulations. Finally, in Section 6 we will conclude.

2. Model of imperfect quantity competition on networks

Before introducing our model of imperfect competition on networks, we first explain perfect competition and then Cournot competition, which is the basic imperfect quantity competition in economics literature. In perfect competition, the price of goods is not determined by firms but by the market where demand and supply are equal. If the price is a bit higher than the market price, then it does not sell at all, whereas if the price is a bit lower than the market price, then the price is lower than the cost and a firm incurs loss as it sells. On the other hand, a firm determines the price in imperfect competition.

In the present paper, we will study quantity imperfect competition. In quantity competition, a firm chooses the amount of goods to sell in the market. Since the price is a function of the quantity in the market, a firm has the power to determine the price through the quantity. A firm determines the price so that the profit is maximized.

Let us explain Cournot competition, in which all firms interact with all the other firms through the market. Suppose that n firms produce the same goods and the firm i produces the amount x_i, which is what a firm can decide. The price p is determined by the total amount X in the market:

$$X = \sum_{i=1}^{n} x_i. \tag{1}$$

The function that gives the price $p(X)$ is an inverse of the function of demand $X(p)$ determined by the price. We hence refer to $p(X)$ as the inverse demand function. We hereafter assume the linear form

$$p(X) = a - dX, \tag{2}$$

where a and d are parameters. The parameter d represents how much the demand is decreased if the price goes up.The parameter a must be large enough for the price to be positive. The profit π_i of firm i is given by

$$\pi_i = p(X)x_i - cx_i, \tag{3}$$

where c is the parameter for unit cost. A firm chooses the amount of goods to produce in order to maximize the profit:

$$x_i = \operatorname{argmax}_{x_i} \pi_i. \tag{4}$$

Since all firms solve the same problem, the amount of goods produced by each firm is equal. Let us denote the amount by x. The solution is called the Cournot–Nash solution, which is given by

$$x = \frac{a - c}{d(1 + n)} \tag{5}$$

$$p = \frac{a + cn}{1 + n}, \tag{6}$$

where we assume that $a - c > 0$ holds.

2.1. Competition on networks

We now consider imperfect quantity competition on networks; see Fig. 1. We will extend imperfect quantity competition to one with a network structure. The vertices represent firms and the network structure represents rivalry relationship among firms. Each firm produces only one kind of different goods. Let k_j denote the degree of the vertex (the firm) j. Let ∂j denote all vertices neighboring vertex j. We assume that the price p_j of the firm j's goods is affected by the amount of goods