Applied Mathematical Modelling 33 (2009) 3002–3014

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modelling the depletion of forestry resources by population and population pressure augmented industrialization

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ARTICLE INFO

Article history:
Received 2 March 2007
Received in revised form 27 September 2008
Accepted 13 October 2008
Available online 30 October 2008

Keywords:
Resource biomass
Population pressure
Industrialization
Stability

ABSTRACT

In this paper, a nonlinear mathematical model is proposed and analysed to study the depletion of forestry resources caused by population and population pressure augmented industrialization. It is shown that the equilibrium density of resource biomass decreases as the equilibrium densities of population and industrialization increase. It is found that even if the growth of population (whether intrinsic or by migration) is only partially dependent on resource, still the resource biomass is doomed to extinction due to large population pressure augmented industrialization. It is noted that for sustained industrialization, control measures on its growth are required to maintain the ecological stability.

1. Introduction

The main causes for the depletion of forestry resources in the developing countries are their use by ever growing population for infrastructure and industrial development [1–15]. The use of forestry resources by human population is done generally in two ways. Firstly, the human population uses forestry resources for its intrinsic growth in the form of fuel, fodder for cattle required for milk production, medicine, etc. directly by cutting trees, plants, herbs, grasses, etc. without clearing the forest land. Secondly, for the development of infrastructure, forest stands are cut in large segments to construct farm houses, cattle farms, housing colonies, health and recreation centers, to set up industrial units, to use land for agriculture, etc. The number of such projects related to development increase immensely as population density increases leading to augmentation of industrialization. If the focus is on the wood based industries, forest trees are used for manufacturing logs, planks, wooden tiles, furniture etc. by cutting forest stands. This kind of situation exists in the Doon Valley located in the northern India (see Ref. [13] and cross-references therein) where population increases due to intrinsic growth and migration.

It may be noted here that due to excessive increase in population density many kinds of precursors, both social and environmental, appear in the habitat. One environmental precursor is pollution, the effects of which on forest resources have been studied by many investigators [1,12]. The other is population pressure which is caused by excessive increase in population density in and around industrial units in forest habitat leading to augmentation of industrialization.

To the best of our knowledge, the effect of population pressure on depletion of forest resources by augmenting industrialization has not been studied. Our focus in this paper is to study this aspect with help of a mathematical model. It has been

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doi:10.1016/j.apm.2008.10.028

shown by various modelling studies that when resources are depleted by increase in population, pollution and industrialization, then the ecological balance of the area under consideration is threatened [13–15]. In particular, Shukla et al. [13] proposed and analysed a mathematical model to study the effect of industrialization (population) pressure on resource depletion showing that the resource may be driven to extinction if the rate of industrialization (population) increases without control. Shukla et al. [14] have further investigated the effect of changing habitat on survival of species caused by industrialization (population). Shukla and Dubey [15] have also studied the detrimental effects of pollution and population on forestry resources by proposing a nonlinear mathematical model. It has been shown that the combined effects of these two factors are more severe on the depletion of forestry resources in comparison to the case when only single factor is involved. It may be pointed out here that in above investigations the depletion of resources caused by population pressure augmented industrialization has not been studied. Therefore, in this paper, a nonlinear mathematical model is proposed and analysed to study the depletion of forestry resources by population pressure augmented industrialization. Our aim is to show that even if the population growth (caused by intrinsic growth or migration) is only partially dependent on resource, the resource biomass is doomed to extinction caused by increase in population density and population pressure augmented industrialization.

2. Mathematical model

Consider a forest habitat where trees are cut by wood industries regularly causing depletion of forest stands. It is assumed that human population in the habitat cut trees for fuel, fodder, housing, expansion of agricultural land, leading to further depletion of forest resources. Thus, in this paper, we model the depletion of renewable (biological) resources caused by population and industrialization, the density of which is augmented by population pressure. It is assumed that the growth rate of population pressure is proportional to the density of population. It is further assumed that growth rate of cumulative biomass density of forest resources is depleted by population and industrialization simultaneously. In view of above, a model governing the dynamics of the system under consideration is proposed as follows.

$$
\begin{aligned}
\frac{dB}{dt} &= s\left(1-\frac{B}{L}\right)B - s_0B - \beta_2NB - s_1IB,\\
\frac{dN}{dt} &= Q(N) - r_0N + \beta_1NB,\\
\frac{dP}{dt} &= \lambda N - \lambda_0P - \theta P,\\
\frac{dI}{dt} &= \pi\theta P + \pi_1s_1IB - \theta_0I,\\
N(0) &= N_0 \geqslant 0,\quad B(0)=B_0\geqslant 0,\quad P(0)=P_0\geqslant 0,\quad I(0)=I_0\geqslant 0,\quad 0\leqslant\pi\leqslant 1,\quad 0\leqslant\pi_1\leqslant 1.
\end{aligned} \tag{1}
$$

In model (1), $B(t)$ is the cumulative density of biomass of resources with intrinsic growth rate coefficient s and carrying capacity L, $N(t)$ is the cumulative density of populations (both human and cattle), $P(t)$ is the density of population pressure caused by increase in cumulative density of populations, and $I(t)$ is the density of industrialization. Further, $s_0 > 0$ is the natural depletion rate coefficient of resource biomass, $r_0 > 0$ is the natural depletion rate coefficient of population, $\beta_1 > 0$ is the growth rate of cumulative density of populations due to resources and $\beta_2 > 0$ is its corresponding depletion rate coefficient of the resource biomass density due to population, $\lambda > 0$ is the growth rate coefficient of population pressure, $\lambda_0 > 0$ is its natural depletion rate coefficient and $\theta > 0$ is its depletion rate coefficient caused in augmenting industrialization, $s_1 > 0$ is the depletion rate coefficient of the biomass density caused by industrialization. The coefficient π_1s_1 is the growth rate of industrialization due to resource, π is the growth rate of industrialization due to population pressure, and $\theta_0 > 0$ is control rate coefficient of industrialization due to external measures applied by Government agencies.

In the second equation of model (1), $Q(N)$ denotes the growth function of the population which is assumed to be of the following two types:

(i) $Q(N) = r(1-\frac{N}{K})N$. In such a case the population is growing logistically with intrinsic growth rate r and carrying capacity K.
(ii) $Q(N) = Q_0$, a constant. In this case, the constant Q_0 denotes the growth of population due to migration in the habitat.

In the following we analyse model (1) using stability theory of differential equations [16].

Case I: Effect of logistic growth of population, that is, $Q(N) = r(1-\frac{N}{K})N$.

First of all, in the following lemma we show that all solutions of model (1) are nonnegative and bounded.

Lemma 1. *The set* $\Omega = \{(B,N,P,I): 0\leqslant B\leqslant L, 0\leqslant N\leqslant N_m, 0\leqslant P\leqslant P_m, 0\leqslant B+P+I\leqslant(\lambda N_m + sL)/\phi\}$ *attracts all solutions initiating in the interior of the positive orthant, where*

$$N_m = \frac{K}{r}(r+\beta_1L),\quad P_m = \frac{\lambda N_m}{\lambda_0+\theta},\quad \phi = \min(s_0,\lambda_0,\theta_0).$$