Nonlinear Analysis: Real World Applications 11 (2010) 2833–2848

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Modelling effects of industrialization, population and pollution on a renewable resource

B. Dubey *, A.S. Narayanan

Mathematics Group, Birla Institute of Technology and Science, Pilani – 333 031, India

ARTICLE INFO

Article history:
Received 7 March 2009
Accepted 6 October 2009

Keywords:
Resource
Industrialization
Population
Pollution
Stability

ABSTRACT

In this paper, a mathematical model is proposed and analysed to study the simultaneous effect of industrialization, population and pollution on the depletion of a renewable resource. Criteria for local stability, global stability and instability are obtained. It is shown that if the densities of industrialization, population and pollution increase, then the density of the resource biomass decreases and it settles down at its equilibrium level whose magnitude is lower than its original carrying capacity. It is further noted that if these factors increase unabatedly, the resource biomass may be driven to extinction. Computer simulations are also performed to illustrate the results.

1. Introduction

One of the important problems that our modern society faces today is the depletion of resources such as forestry, fishery, fertile topsoil, crude oil, minerals, etc. It is a well known accepted fact that resource plays a significant role in the development of any country. But it is being depleted by increased industrialization, over growth of population (particularly in the third world countries) and associated pollution. A typical example is the Doon Valley in the northern part of India where the forestry resources are being depleted by limestone quarries, wood and paper based industries, growth of human and livestock populations, expansion of forest land for agriculture and settlement, etc., threatening the ecological stability of the entire region [1–3]. It has been noted that the forest biodiversity loss and changes in climate are closely linked with deforestation [4–7].

In recent years several investigations were made to study the effect of toxicant/pollutant on various ecosystems using mathematical models [8–17]. Hallam and his co-workers [12–14] have studied the crucial role played by toxicant uptaken by biological species. Freedman and Shukla [10] presented a mathematical model to study the effect of a toxicant on a single-species population and predator–prey system and investigated the effect of changes in the carrying capacity of the habitat caused by the presence of toxicant in the environment. Hauping and Zhein [15] also proposed a model to study the effect of a toxicant on two interacting biological species by considering that the two species have the same uptake concentration. Shukla et al. [2] studied the growth and survival of resource biomass dependent species in a forested habitat which is being depleted due to the pressure of industrialization. Shukla and Dubey [18] described a mathematical model to study the simultaneous effects of two toxicants with different toxicities. Thomas et al. [19] studied the effect of environmental pollution on a single-species population and derived some criteria to restrict the amount of pollution in the environment to ensure the survival of the population. Shukla and Dubey [20] also modelled the depletion of a forestry resource in a habitat, which is caused by increases in population density and pollutant emission into the environment. Hsu and Waltman [21] studied the effect of a toxicant on two competing species by assuming that the toxicant is produced by one of the competitors. Buonomo et al. [22] obtained some threshold results for the survival of a population living in a polluted environment. Dubey and Das [23] proposed a model to study the effect of industrialization on growth and existence

* Corresponding author. Tel.: +91 1596 243227; fax: +91 1596 244183.
E-mail addresses: bdubey@bits-pilani.ac.in, balram.dubey@gmail.com (B. Dubey).

1468-1218/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2009.10.007

of a biological species that depends partially or wholly on a given resource or act as a predator on the resource. Dubey and Hussain [24] studied the effect of a pollutant on the dynamics of two species, in which interaction is taken as competition, cooperation and predation, by considering that the two species have different uptake concentration. Dubey and Hussain [25] further proposed a model to study the coexistence of two competing plant species in a finite habitat where each species produces a toxicant affecting the other species. Hsu et al. [26] also studied the competition between two species competing for a nutrient in the presence of a toxicant, which is lethal to one competitor but being taken by the other without any harm. Shangbing [27] further improved the results obtained in [26] and derived necessary and sufficient conditions for the existence of a unique positive equilibrium point and a set of sufficient conditions for the existence of periodic solutions for a 3-dimensional system which arises from a model of competition between plasmid-bearing and plasmid-free organisms in a chemostat with an inhibitor or a toxicant. Shukla et al. [28] proposed a mathematical model to study the effect of a toxicant, which is emitted into the environment from external sources, on two competing species. They showed that the four usual outcomes of competition between two species may be altered under appropriate conditions which are mainly dependent on emission rate of toxicant into the environment, uptake concentration of toxicant by two species and their growth rate coefficients and carrying capacities. Dubey et al. [29] studied the depletion of a resource biomass due to industrialization and pollution. It has been shown here that in the case of small periodic influx of pollutant into the environment, the resource biomass has a periodic behaviour if the depletion rate coefficient of environmental pollutant is small. However, if this coefficient increases beyond a threshold value, then the resource biomass converges towards its equilibrium. Dubey and Hussain [30] proposed a mathematical model and studied the effect of environmental pollution on forestry resources with time delay in a diffusive system. It has been found here that if the depletion rate of environmental pollutant due to its uptake by resource biomass is high, then it destabilizes the system. But low rate of formation of toxic substance due to some metabolic changes inside the resource biomass stabilizes the system. Dubey and Hussain [31] further studied the survival of a biological species which is dependent upon a resource and living in a polluted environment. They observed that the density of population is maximum when the population depends partially on the resource, and minimum when the populations is predating on the resource. Consequently, the density of the resource biomass is minimum in the partially dependent case and maximum in the predation case. Recently, Dubey et al. [32] proposed a nonlinear mathematical model to study the depletion of forestry resources caused by population and population pressure augmented industrialization. It has been noted here that for sustained industrialization, control measures on its growth are required to maintain the ecological stability. Shukla et al. [33] investigated effects of toxicants emitted from external sources as well as from its precursors on the survival of a resource dependent species.

It may be pointed out here that in the above investigations, individual effect of each of the factors such as industrialization, population and pollution or the joint effects of population and pollution have been studied. However, the combined effects of industrialization, population and pollution have not been investigated. Keeping these in view, in this paper we propose and analyse a mathematical model to understand the dynamics of depletion of a renewable resource due to the pressures of industrialization, population and pollution.

2. The model

We consider an ecosystem where the resource biomass is being continuously depleted due to the pressures of industrialization, population and pollution. It is assumed that the dynamics of the resource biomass, industrialization and population densities are governed by logistic type equations. It is also assumed that the growth rate of the resource biomass decreases with increase in the uptake concentration of pollutant, industrialization and population densities while its carrying capacity decreases with increase in environmental concentration of pollutant, industrialization and population densities. It is further assumed that the growth rate of population density increases as the density of the resource biomass increases. It is also considered that industrialization and population may enhance the growth of each other. It is further assumed that the emission of pollutant into the environment is industrialization and population dependent. Following Freedman and Shukla [10] and Shukla and Dubey [20], the dynamics of the system may be governed by the following differential equations:

$$
\begin{aligned}
\frac{dB}{dt} &= r_B(U,P)B - \frac{r_{B0}B^2}{K_B(I,P,T)} - \alpha IB, \\
\frac{dI}{dt} &= r_I I\left(1 - \frac{I}{L}\right) + \beta IB + \gamma_1 IP, \\
\frac{dP}{dt} &= r_P(B)P - \frac{r_{P0}P^2}{M} + \gamma_2 IP, \\
\frac{dT}{dt} &= Q(I,P) - \delta_0 T - \alpha_1 BT + \pi\nu BU + \theta_1\delta_1 U, \\
\frac{dU}{dt} &= -\delta_1 U + \alpha_1 BT - \nu BU + \theta_0\delta_0 T, \\
& B(0) \geq 0,\ I(0) \geq 0,\ P(0) \geq 0,\ T(0) \geq 0,\ U(0) \geq 0,\ 0 \leq \pi, \theta_0, \theta_1 \leq 1.
\end{aligned}
\tag{2.1}
$$