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The returns and risks of investment portfolio in a financial market ☆

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HIGHLIGHTS

- The model of the investment portfolio in financial market was established.
- The agreements between the results of our model and Dow Jones Industrial Average were found.
- The effects of dispersion of investment portfolio are analyzed.
- The roles of investment period on risks and returns are discussed.

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ABSTRACT

The returns and risks of investment portfolio in a financial system was investigated by constructing a theoretical model based *on the Heston model*. After the theoretical model and analysis of portfolio were calculated and analyzed, we find the following: (i) The statistical properties (i.e., the probability distribution, the variance and loss rate of equity portfolio return) between simulation results of the theoretical model and the real financial data obtained from Dow Jones Industrial *Average are in good* agreement; (ii) The maximum *dispersion of the investment* portfolio is associated with the maximum stability of the equity portfolio return and minimal investment risks; (iii) An *increase of the investment* period and a worst investment period are associated with a decrease of stability of the equity portfolio return and a maximum investment risk, respectively.

1. Introduction

The researches of financial markets as complex systems in statistical physics have obtained more and more attentions and given rise to a new field called “econophysics” in recent years [1,2]. The geometric Brownian motion *model was early* proposed to describe the stochastic dynamics of stock prices [3,4]. However, this geometric Brownian motion model cannot agree with some statistical characteristics from actual financial data, such as, the fat tails [5,6], long range memory and clustering of volatility [7]. Afterwards, many valuable *models have been developed* to make up these deficiencies, such as the ARCH model [8], GARCH model [9], and Heston model [10]. In particular, the Heston model *is well consistent with* statistical characteristics of stock prices obtained from actual financial data, for example, the probability distribution of returns for Dow-Jones data [11] and for the three major stock-market indexes (Nasdaq, S&P500, and Dow-Jones) [12], the exponential distribution of financial returns obtained from actual financial data [13], the probability density distribution of the logarithmic returns of the empirical high-frequency data of DAX and its stocks [14] or the typical price fluctuations of the Brazilian

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São Paulo Stock Exchange Index [15]. Meanwhile, the Heston model has been widely used to analyze dynamics of stock price in actual financial markets, for example, the mean escape time in a modified Heston model with a cubic nonlinearity [16,17], the statistical properties of the hitting times in different models for stock market evolution [18,19], exact expressions for the survival probability and the mean exit time [20,21], the effects of the delay time in Heston model [22,23], and the stochastic resonance of the stock prices [24].

In the actual trade in financial market, rational investors try to choose an appropriate investment portfolio, due to lower investment risks and higher stability of returns. Since Merton [25] used a stochastic optimal control approach of geometric Brownian motion model to analyze the investment optimization, investment portfolio has been discussed in an incomplete semimartingale market by Kramkov and Schachermayer [26,27] and Bouchard et al. [28], in accounting by Kothari [29], in risk-based indifference pricing [30] and so on. Hence, investment portfolio need to be further investigated in actual market. Even though portfolio with Heston model is discussed in some studies, such as bond portfolio selection problem [31] and financial planning [32], the studies are deficient in dispersion of investment portfolio, investment period and analysis of real financial data, etc. In order to make up the deficiencies, we use the Heston model to construct a model for the investment portfolio due to the model more close to reality.

In this paper, we establish a model for investment portfolio with the Heston model to discuss the returns and risks of investment portfolio. In addition, the remainder of this paper is organized as follows. In Section 2, a model for investment portfolio is introduced with the Heston model. In Section 3, the returns and risks are discussed in a investment portfolio of two uncorrelated stocks. In Section 4, a brief conclusion ends the paper.

2. The investment portfolio with the Heston model

In the actual financial investment, equity portfolio consists of the price and quantity of n stocks held by investors, and then we define the equity portfolio $S(t)$ as

$$S(t) = \sum_{i=1}^{n} n_i(t) S_i(t), \tag{1}$$

where at time t, $S_i(t)$ is the price of the ith stock and $n_i(t)$ is the quantity of the ith stock ($i = 1, 2, 3 \ldots n$), and then the rate of equity portfolio $C(t)$ can be written as:

$$\begin{aligned} C(t) &= \frac{S(t)}{S(0)} \\ &= \frac{\sum\limits_{i=1}^{n} n_i(t) S_i(t)}{S(0)}. \end{aligned} \tag{2}$$

Let us define the initial rate of the i-th stock on the total equity portfolio

$$r_i = \frac{n_i(0) S_i(0)}{S(0)}. \tag{3}$$

From Eqs. (2) and (3), we can find $C(0) = \sum_{i=1}^{n} r_i = 1$. For $\frac{dn_i(t)}{\mathrm{d}t} = 0$, fixing the logarithmic stock price $x_i(t) = \ln S_i(t)$, and employing the Heston model [10,11] to describe the stock price *dynamics,* Eq. (2) *becomes*

$$C(t) = \sum_{i=1}^{n} r_i \exp(x_i(t) - x_i(0)), \tag{4}$$

where

$$\mathrm{d}x_i(t) = \left(\mu_i - \frac{\nu_i(t)}{2}\right) \mathrm{d}t + \sqrt{\nu_i(t)}\, \mathrm{d}W_i(t), \tag{5}$$

and

$$\mathrm{d}\nu_i(t) = a_i(b_i - \nu_i(t))\, \mathrm{d}t + c_i \sqrt{\nu_i(t)}\, \mathrm{d}Z_i(t). \tag{6}$$

Here the subscript i indicates the ith stock, μ_i is the growth rate, $\nu(t)_i$ is the volatility of the stock price, a_i is the mean reversion of the $\nu(t)_i$, b_i is the long-run variance, c_i is *the volatility of volatility, that is* the amplitude of volatility fluctuations [33], $\mathrm{d}W_i(t)$ and $\mathrm{d}Z_i(t)$ are correlated *Wiener processes with the following statistical properties*

$$\begin{aligned} &\langle \mathrm{d}W_i(t) \rangle = \langle \mathrm{d}Z_i(t) \rangle = 0, \\ &\langle \mathrm{d}W_i(t)\, \mathrm{d}W_j(t') \rangle = \langle \mathrm{d}Z_i(t)\, \mathrm{d}Z_j(t') \rangle = \delta_{i,j} \delta(t - t')\, \mathrm{d}t, \\ &\langle \mathrm{d}W_i(t)\, \mathrm{d}Z_j(t') \rangle = \rho_{i,j} \delta(t - t')\, \mathrm{d}t, \\ &\langle \mathrm{d}Z_i(t') d_j W(t) \rangle = \lambda_{i,j} \delta(t - t')\, \mathrm{d}t, \end{aligned} \tag{7}$$

$\rho_{i,j}$ denotes the cross correlation coefficient between $\mathrm{d}W_i(t)$ and $\mathrm{d}Z_j(t)$, and $\lambda_{i,j}$ denotes the cross correlation coefficient between $\mathrm{d}Z_i(t)$ and $\mathrm{d}W_j(t)$.