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The veto mechanism in atomic differential information economies[☆]

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ABSTRACT

We establish new characterizations of Walrasian expectations equilibria based on the veto mechanism in the framework of differential information economies with a complete finite measure space of agents. We show that it is enough to consider the veto power of a single coalition, consisting of the entire set of agents, to obtain the Aubin private core. Moreover, we investigate on the veto power of arbitrarily small and big coalitions, providing an extension to mixed markets of well known Schmeidler (1972) and Vind's (1972) results in terms of Aubin private core allocations.

1. Introduction

The aim of this paper is to investigate on the veto mechanism in differential information economies with a finite number of states of nature and a measure space of agents that may have atoms, when some restrictions on admissible coalitions are imposed. From the mathematical point of view, an atom is a subset of the space of agents with strictly positive measure containing no proper subsets with strictly positive measure and it is typically used to represent an economic individual concentrating in his hands a large initial ownership compared with the total market endowment. Even if the initial resources are spread over a continuum of small traders, it could be the case that some of them decide to act only together, as a single individual, without the possibility to form proper subgroups. This scenario, still represented via atoms, includes cartels, syndicates and other forms of institutional agreements. It is well known that the presence of non negligible traders causes a lack of perfect competition and consequently the failure in the Core–Walras Equivalence Theorem. Nonetheless, it is sometimes possible to extend the core equivalence theorem to mixed markets (see Greenberg and Shitovitz, 1986, Pesce, 2010 and Shitovitz, 1973 among others for contributions in this direction), paying a cost in terms of assumptions, since it is needed that large traders lose their market power becoming competitors. This is guaranteed assuming, as Shitovitz suggested (Shitovitz, 1973; see also De Simone and Graziano, 2003 for an extension to infinite dimensional commodity spaces and Pesce, 2010 for an extension to differential information economies), that there are at least two large traders of the same type, meaning that they have the same initial endowment and same preferences. In order to characterize competitive equilibrium allocations without imposing any additional conditions on the atomic sector, we consider the Aubin approach to core analysis (see Aubin, 1979), according to which agents may participate by using only a fraction of their initial resources when forming a coalition. This new pondered veto concept was introduced by Aubin (1979) in complete information economies with a finite number of agents and commodities, in order to characterize the set of competitive equilibria, when the ordinary core is too large to coincide with it. Later, Noguchi (2000) proved that even in the presence of atoms, the Aubin core provides a characterization of competitive equilibria. The Aubin pondered veto concept and the equivalence with the set of Walrasian equilibria were extended in the framework of atomic differential information economies with a finite number of states of nature by Graziano and Meo (2005) (see also Pesce, 2010 in which the free disposal condition is avoided and Basile et al. (2012) for the case of atomic economies with public goods). Under uncertainty and with asymmetrically

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informed agents, it has been also required that members of a blocking coalition can only use their own private information, since they have no opportunities to share information.

Checking whether a given allocation belongs to the Aubin private core seems to require to look upon the whole set of possible coalitions in order to test whether any groups of agents, by employing a rate of their own initial endowment and by using their own private information, can improve upon such allocation. Therefore, this seems to be hard to check, unless the economy is very small. As pointed out by Hervés-Beloso and Moreno-García (2001), it may be difficult to argue that coalition formation is costless and free: "*The fact that agents are organized in some way, and perhaps they are not entirely free, may result in high formation costs, commitments and constraints, which make difficult to assume that the veto mechanism works freely and spontaneously*". For this reason, it is usually assumed that only a subset of the set of all possible coalitions in an economy is considered to be really formed. The papers (Hervés-Beloso and Moreno-García, 2001, 2008; Hervés-Beloso et al., 2005a) go in this direction (see also Hervés-Beloso et al., 2005b for an infinite dimensional commodity space setting). They obtain a characterization of Walrasian equilibria which differs substantially from the equivalences obtained by Debreu and Scarf (1963) and Aubin (1979). Indeed, on the one hand Debreu–Scarf and Aubin enlarge the set of blocking coalitions: the former by replicating the economy, the latter by allowing the participation of the agents with any rate of their endowments. On the other hand, Hervés-Beloso and Moreno-García (2001), Hervés-Beloso and Moreno-García (2008) and Hervés-Beloso et al. (2005a) consider the veto power of just one coalition, namely the grand coalition, by enlarging the possible redistribution of endowments. In other words, they consider the veto power of a single coalition in infinitely many economies obtained by perturbing the original initial endowments.

In this paper, we extend their result by showing that the Aubin private core coincides with the set of those allocations which are not privately blocked by any generalized coalition with full support, that is whose support equals the set of all agents. Our result differs from Hervés-Beloso et al. (2005a) in two main aspects: first we slack the assumption of "finitely many agents" by considering the general case of differential information mixed markets. Second, we do not need to construct a family of economies perturbing agents' initial endowment, since we show that it is enough to consider the pondered veto power of a single coalition in just one economy.

Our analysis on the implications that restrictions on the measure of a blocking coalition may have on the Aubin private core proceeds by extending Schmeidler and Vind's Theorems to the set of feasible allocations not privately blocked by any generalized coalition in mixed differential information economies. Schmeidler and Vind's Theorems performed in a new way the Core–Walras equivalence theorem obtained by Aumann in 1964 (Aumann, 1964) for atomless economies. Schmeidler (1972) showed that it is enough to consider the veto power of arbitrarily small coalitions to get the core, and Vind (1972) that any allocation, not blocked by arbitrarily large coalitions, is in the core. In both results the hypothesis that the economy is atomless is crucial. The aim of this paper is to provide conditions guaranteeing that in mixed economies with asymmetrically informed agents, given any positive number α, less than the measure of the grand coalition, an allocation outside the Aubin private core can be blocked by a generalized coalition whose support has measure smaller than α (extension of Schmeidler's Theorem) and by a generalized coalition whose support has measure equal to α (extension of Vind's Theorem). If the economy is atomless, then Schmeidler and Vind's Theorems easily follow from Lyapunov convexity theorem; but, if there are some large traders, it could be not possible to reduce the measure of a blocking coalition as much as we want. That is why in the case of mixed differential information economies a restriction on the real number α is needed. Indeed, in atomless economy the Aubin private core does not change whatever restriction on the measure of a blocking coalition is imposed; while in mixed markets an allocation outside the Aubin private core can be privately blocked only by generalized coalitions whose support has a measure smaller than (or equal to) any α greater than the measure of the atomic sector. Whenever α is smaller than the measure of the atomic sector, we need to make negligible the veto power of large traders in order to manage the measure of a blocking coalition. We show in Example 4.1 that for our purpose the presence of at least two atoms of the same type, according to Shitovitz's assumption, may not be enough, and a stronger hypothesis on the atomic sector is needed. We prove that if there are countably many large traders of the same type, even in a mixed market, an allocation x outside the Aubin private core is privately blocked by a generalized coalition whose support has arbitrarily small measure (Theorem 4.3) and by a generalized coalition such that the measure of its support is smaller than the measure of the atomless sector (Theorem 4.4). To this end the allocation x must satisfy what we call the "equal treatment property on the atomic sector", according to which identical large traders are equally treated under x. We also illustrate some examples to underline the necessity of the hypotheses used and establish a list of new characterizations of Walrasian expectations allocations based on the veto mechanism.

The paper is organized as follows. We first present the theoretical model and state main definitions; then, in Section 3 we investigate on the veto mechanism of the grand coalition while Section 4 contains extensions to mixed markets of Schmeidler and Vind's results in terms of Aubin private core allocations. Proofs are collected in the Appendix.

2. The model and the main definitions

In this section we illustrate the theoretical framework for studying exchange economies with uncertainty and asymmetrically informed agents. First, we formally present the basic model describing briefly each component of it and then we focus on the key solution concepts that we will use throughout our analysis.

2.1. The model

We consider a Radner-type exchange economy $\mathcal{E}$ with differential information, modeled by the following collection:

$$\mathcal{E} = \left\{(\Omega, \mathcal{F});\ (T, \mathcal{T}, \mu);\ \mathbb{R}^{\ell}_{+};\ (\mathcal{F}_t, q_t, u_t, e_t)_{t\in T}\right\}$$

where:

1. $(\Omega, \mathcal{F})$ is a measurable space describing the exogenous uncertainty; Ω is the finite set denoting the possible states of nature (i.e., $\Omega = \{\omega_1, \ldots, \omega_k\}$) and $\mathcal{F}$ is the field of all the events (i.e., the power set of Ω).
2. $(T, \mathcal{T}, \mu)$ is a complete, positive and finite measure space, where: T is the set of agents and $\mathcal{T}$ is the σ-field of all eligible coalitions, whose economic weight on the market is given by the measure μ. A null set of traders is a set of measure 0. A statement asserted for almost all or μ-almost all traders in a certain set is to be understood to hold for all such traders except possibly for a null set of traders. An arbitrary finite measure space of agents makes us deal simultaneously with the case of discrete economies, non-atomic economies as well as economies that may have atoms. Indeed, discrete economies are covered by a finite set T with a counting measure μ. Atomless economies are analyzed by assuming that $(T, \mathcal{T}, \mu)$ is the Lebesgue measure space with $T = [0, 1]$. Finally, mixed markets are those for which T is composed by two sets: T_0 and T_1,