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Dynamics of a Delayed Duopoly Game with Increasing Marginal Costs and Bounded Rationality Strategy

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Abstract

This paper constructs a duopoly game model and investigates its stability with bounded rationality strategy and state delay. Numerical experiments are also carried out to exhibit its dynamical features such as bifurcation diagram, maximal Lyapunov exponent curve, strange attractor and so on. The results show that state delay can enlarge the local stability region of Nash equilibrium point and delay the outputs bifurcation. The faster adjustment speed will induce more complex dynamical response of outputs such as cycles and chaos.

Keywords: Duopoly game; Delay; Bounded rationality; Increasing marginal costs; Numerical simulation

1. Introduction

In recent years, the problem of oligopoly game has been extensively investigated [1-11]. These literatures show that even for the simplest case of duopoly game, the dynamics are extremely complex because one player's profit depends not only on its own quantity decision but also on its rival's reaction. The existing literatures on duopoly game were usually carried out by classical Cournot assumption and three decision-making methods or their combinations: naive method [1-3], adaptive method [4-5] and bounded rationality method [6-11]. In fact, expectation factors play an important role in the progress of duopoly game. Different expectation methods will have great effect on the dynamics of duopoly game.

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For instance, Kamalinejad et al [12] investigated the duopoly game with linear regression expectation. Elsadnay [11] studied the dynamics of a duopoly game with constant marginal cost by introducing the state-delay to estimate the rival's following quantity. However, the marginal cost will never remain static along with the quantity increasing. Also, the information in the market is usually incomplete. Therefore in this paper, we study the dynamics of a duopoly game by using bounded rationality strategy, introducing state delay to estimate the rival's expectation quantity, and considering the increasing marginal costs.

2. Delayed duopoly game model

Consider two Firms in the market producing same or homogeneous goods. The inverse demand function is assumed linear and decreasing: $P = a - bQ$, where $Q = q_1 + q_2$ is the total output and $a, b > 0$. The marginal cost of two Firms is assumed as increasing, i.e., the cost function is of the following form: $C_i(q_i) = c_i + d_i q_i + e_i q_i^2$, $i = 1,2$, where $c_i, d_i, e_i > 0$. In many existing literatures [6-10], depending on the marginal profit in time period t, the player with bounded rationality adjusts its output in time period $t+1$ to maximize its profit. In fact, the maximal profit of Firm i in time period $t+1$ is relevant to the quantity decision of Firm j in time period $t+1$, so we introduce the production expectation $q^e(t+1)$ into the usual bounded rationality model as below [11]:

$$\begin{cases} q_1(t+1) = q_1(t) + v_1 \cdot q_1(t) \cdot \dfrac{\partial \pi_1(q_1(t), q_2^e(t+1))}{\partial q_1(t)} \\ q_2(t+1) = q_2(t) + v_2 \cdot q_2(t) \cdot \dfrac{\partial \pi_2(q_2(t), q_1^e(t+1))}{\partial q_2(t)} \end{cases}, \tag{1}$$

where v_1 and v_2 are positive and represent the relative speed of adjustment. $q_j^e(t+1)$, $j = 1,2$, denotes the expectation of Firm i about the production decision of Firm j. Different from the usual Cournot assumption, the expectation of Firm i about the production decision of Firm j in this paper is expressed by employing state delay, i.e.,

$$q_j^e(t+1) = w_j \cdot q_j(t) + (1-w_j) \cdot q_j(t-1), \quad j = 1,2, \tag{2}$$

where w_j is the weight and satisfies $0 < w_j < 1$. By using (1) and (2), the duopoly game with state delay and bounded rationality can be modeled as following:

$$\begin{cases} q_1(t+1) = q_1(t) + v_1 \cdot q_1(t) \cdot [a - d_1 - 2 \cdot (b+e_1) \cdot q_1(t) - b[w_2 \cdot q_2(t) + (1-w_2) \cdot q_2(t-1)]] \\ q_2(t+1) = q_2(t) + v_2 \cdot q_2(t) \cdot [a - d_2 - 2(b+e_2) \cdot q_2(t) - b[w_1 \cdot q_1(t) + (1-w_1) \cdot q_1(t-1)]] \end{cases}. \tag{3}$$

Equations (3) has only one Nash equilibrium point $E^* = (q_1^*, q_2^*)$, where $q_1^* = \dfrac{2e_2(a-d_1) + b(a + d_2 - 2d_1)}{3b^2 + 4be_1 + 4be_2 + 4e_1e_2}$, $q_2^* = \dfrac{2e_1(a-d_2) + b(a + d_1 - 2d_2)}{3b^2 + 4be_1 + 4be_2 + 4e_1e_2}$. Based on the economic meaning, the parameters must satisfy the following conditions:

$$a > d_1,\ a > d_2,\ a + d_2 - 2d_1 > 0,\ a + d_1 - 2d_2 > 0. \tag{4}$$

Proposition 1. The Nash equilibrium point E^* is locally stable.

Equations (3) include state delay which makes it difficult to analyze the stability of (3). In order to study the stability of Nash equilibrium point E^*, we rewrite (3) as a four-dimensional system in the form: