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Single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments ☆

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ABSTRACT

We consider in this paper an Economic Order Quantity (EOQ) problem involving a single supplier that offers quantity discounts and allows retailers to delay payments. The retailers are tempted to form coalitions in order to minimize their costs. We propose a solution approach that generates stable coalition structures for the retailers taking into account the delay in payments and the discount quantity offered by the supplier. The proposed approach includes a decision rule that generates preferred coalitions for each retailer. Our decision rule reduces considerably the number of explored coalition structures in order to determine solutions in the core.

Through an experimental investigation of the problem, we show the effectiveness of our approach in solving large scale problems and illustrate the effect of permissible delay in payments versus discount quantity on the coalition formation problem.

1. Introduction

In many situations, partners involved in the supply chain (SC) are looking forward minimizing their costs. Suppliers and retailers can cooperate by adopting new purchasing schemes. This is the case when a supplier is offering discounts for the ordering of items. Interestingly, retailers can form coalitions in order to minimize their individual costs. Such cooperation can occur in the ordering, the transportation and the storage of items (Jaber & Zolfaghari, 2008). Li and Huang (1995) discussed various cooperative mechanisms to achieve channel effectiveness: joint ownership, contacts, profit sharing and quantity discounts. Cachon and Lariviere (2001) provided a review on cooperative strategies and coalition formation in the SC. They proved the existence and uniqueness of equilibria in such games. Another form of cooperation in the SC consists in forming coalitions. Slikker, Fransoo, and Wouters (2005) considered a transshipment problem that suggests the coalition formation concept as a powerful tool to minimize the cost of each player. They generated solution structures lying in the core of the game. Özen, Fransoo, Norde, and Slikker (2004) proposed an inventory problem with the possibility of using common warehouses where the coalition formation consists in storing items in the same warehouse. Meca, Timmer, García-Jurado, and Borm (2005) formulated an inventory problem as a cooperative game that minimizes the total cost by grouping the orders to share the payment of the ordering cost.

If the EOQ problem involves only the discount option in the ordering of items, the grand coalition is generally the suitable equilibrium for the retailers. However, other assumptions can prevent the formation of the grand coalition as the delay in payments. Salameh, Abboud, El-Kassar, and Ghattas (2003) studied an inventory model assuming the delay in payment that incurs charged costs. Such procurement problem is noticeable in chemical companies. Suppliers generally propose discounts'landings in terms of the purchased quantity. Therefore, joint orders for numerous companies are profitable. At the same time, the supplier can accept to delay the payment for companies in lack of cash.

We address in this paper an extension of the EOQ problem between a supplier and numerous retailers. The supplier offers a discount quantity and allows the delay in payment to increase the sale. Subsequently, retailers are tempted to form coalitions to benefit from the offered discount. The cooperation between retailers is slowed down by the charged costs incurred by the delay in payment as each retailer proposes his suitable delay period if he faces a lack of cash. This horizontal cooperation can be accurately modeled by a cooperative game that manages the coalition formation process in order to generate solutions in the core. We formulate the EOQ problem assuming these two options for both the supplier

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and each retailer, then show that our model is profitable for both the supplier -as the volume of sales increases- and retailers -since the individual costs decrease-. We propose an algorithm that generates coalition structures lying in the core of the game within a reasonable computational time. Based on a decision rule applied to each retailer, we discard all non profitable coalitions, then merge partial results into a coalition structure that satisfies all standpoints. We show through an empirical investigation the saving in computational time and the fast convergence of our approach. This paper is organized as follows: Section 2 consists in an overview on the discount quantity and the delay options for the EOQ problem. In Section 3, we introduce the main concepts of cooperative game theory. In Section 4, we state a formulation of the cooperative ordering problem with delay in payments. In Section 5, we describe the problem of cooperation between multiple retailers with permissible delay in payments and propose a decision rule that helps each retailer to enumerate his preferred coalitions. These results are performed in the the main algorithm that generated coalition structures in the core. In Section 6, we show the performance of our algorithm in terms of reducing the explored solution space through various computational experiments.

2. An overview on discount and delay in the supply chain

The cooperation for the EOQ problem is designed regarding two main features: the quantity discount and the delay in payment. The existing literature generally addresses a vertical cooperation, inside the SC, between a supplier and a retailer using either of the two cooperation mechanisms. The related literature is summarized in Table 1. The quantity discount, known as a tractable scheme to increase the sale volume, is studied with numerous variations. For the delay, we can point out two main forms: with charged cost and without. We outline in the subsequent sections these two mechanisms and explain the addressed corresponding problems.

2.1. Discount quantity

Chen, Federgruen, and Zheng (2001) addressed a distribution channel where a supplier delivers a single product to multiple retailers. They proposed a new discount scheme for annual sales volume, order quantity and order frequency. They showed that no classical discount scheme related to the ordered quantities suffices to optimize channel profits when there are multiple nonidentical retailers. Mendoza and Ventura (2008) incorporated the discount quantity option in the EOQ including two different modes of transportation. They considered a single stage model with all-units and incremental quantity discount structures. They computed optimal inventory policies in terms of the reorder interval.

Table 1
EOQ papers evoking discount quantity or delay in payments.

Options\between	One supplier and one retailer	One supplier and multiple retailers
Discount quantities	Leopoldo and Cardenas (2009) Mendoza and Ventura (2008) Shin and Benton (2007) Qin et al. (2007), Jaber et al. (2006) Sarker and Al Kindi (2006) Jianli Li and Liu (2006)	Meca et al. (2005) Chen et al. (2001)
Delay in payments	Jaggi Chandra et al. (2008) Chen and Kang (2007) Jaber and Osman (2006), Huang (2005) Teng et al. (2005), Salameh et al. (2003) Chang et al. (2003), Goyal (1985)	Chi Kin Chan et al. (2010)

Jaber, Osman, and Guiffrida (2006) proposed a three-level (supplier–manufacturer–retailer) supply chain model with quantity discount and investigated two different profit sharing scenarios to maximize the supply chain profit. They showed that as demand becomes more sensitive to price discount, the replenishment policies for the players were to order in larger quantities. More recently, Leopoldo and Cardenas (2009) proposed a corrected model of Sarker and Al Kindi (2006). They modeled an EOQ with discounted prices in order to obtain the optimal ordering policy during a special discount price period. Different scenarios of the relation of sale period with the replenishment time were presented. They rectified the mathematical formulation of Sarker and Al Kindi (2006) and determined the optimal ordering policies. Benton and Park (1996) presented an overview of the quantity discount. They classified the literature under various types of discount schemes as all-unit and incremental discounts. Numerous papers studied such EOQ problem with the discount option as Shin and Benton (2007), Jianli Li and Liu (2006), Qin, Tang, and Guo (2007) Goossens, Maas, Spieksma, and Van de Klundert (2007). In our paper, we consider the quantity discount as an incentive of the coalition formation in which case grouping orders is profitable for both the supplier and retailers.

2.2. Delay in payments

In opposition to the traditional EOQ models where the payment is made immediately, numerous researches considered the delay in payments option as an incentive to increase the sale volume. Goyal (1985) was the first to propose an EOQ model under permissible delay in payments. He considered the retailer's inventory replenishment problem under trade credit period.

Salameh et al. (2003) assumed that if the retailer chooses to delay payments by lack of cash, some interests are to be charged with an annual rate. They determined the optimal ordering quantity and reorder level that maximize the vendor's total expected profit per unit time when payments are delayed. A fully permissible delay in payments was considered by Chang, Ouyang, and Tend (2003). They assumed that the delay in payments is allowed by the supplier only if the ordered quantity is greater than a predetermined threshold. Teng, Chang, and Goyal (2005) modeled a EOQ problem with permissible delay in payments and develop an algorithm to determine the optimal price for a retailer and his lot size simultaneously. Huang (2005) investigated the retailer's optimal replenishment policy under permissible delay in payments and introduced the notion of partially permissible delay in payments in opposition to the fully permissible delay in payments. An integrated inventory model with permissible delay in payments was developed by Chen and Kang (2007), where they proposed a cost allocation algorithm to determine the solution. Jaber and Osman (2006) presented a mathematical model for a supplier-retailer coordination with permissible delay in payments and proposed a profit sharing scenario. They modeled the delay period as a decision variable. Recently, Chi Kin Chan, Lee, and Goyal (2010) showed through a single vendor a multi-buyer supply chain that considering delay in payments improves the total chain cost. They proposed a profit-sharing scheme to guarantee an equitable sharing of the model saving. Jaggi Chandra, Goyal, and Goel (2008) developed an alternative inventory model under two levels of trade credit policy. They assumed that the demand is linked to the credit period offered for the customers. In these models, the demand is formulated in terms of the credit period offered to customers with delay in payments.

In this paper, we consider an all-unit discount quantity with delay payment period where an additional cost will be charged over the delay period. We assume that the delay period is a retailer's decision variable in the time spam $[1, T]$. Delay in payments incurs an additional interest cost for retailers (Salameh et al., 2003) and does not encourage them to order jointly their quantities.