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Multi-product budget-constrained acquisition and pricing with uncertain demand and supplier quantity discounts

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ABSTRACT

We consider the joint acquisition and pricing problem where the retailer sells multiple products with uncertain demands and the suppliers provide all unit quantity discounts. The problem is to determine the optimal acquisition quantities and selling prices so as to maximize the retailer's expected profit, subject to a budget constraint. This is the first extension to consider supplier discounts in the constrained multi-product newsvendor pricing problem. We establish a mixed integer nonlinear programming (MINLP) model to formulate the problem, and develop a Lagrangian-based solution approach. Computational results for the test problems involving up to thousand products are reported, which show that the proposed approach can obtain high quality solutions in a very short time.

1. Introduction

Due to demand uncertainty, the matching of supply and demand is a constant challenge faced by a retailer. Product acquisition and pricing are used as two levers in the retailer's up-stream and down-stream to better match supply and demand. A retailer can use pricing to manage demand and increase the revenue, and optimize acquisition quantity or inventory level to reduce the mismatch and cost by exploiting economies of scale. How to integrate both pricing and acquisition decisions under uncertain demand is a challenging problem. The situation becomes more complicated when suppliers provide quantity discounts: the retailer can procure products at a lower unit price if the acquisition quantity is over a certain value—the threshold; however, since the demand is uncertain, the retailer's over-stocking risk will increase. Through setting a suitable price, the retailer can reduce overstocking risk and increase revenue. Thus coordinating the acquisition decision and pricing with uncertain demands becomes more practical and challenging when suppliers offer quantity discounts. Motivated by the observation, this research investigates the joint acquisition and pricing problem with uncertain demand and supplier discounts. The problem is to determine the optimal ordering quantities and selling prices simultaneously so as to maximize the retailer's expected profit.

The problem is an extension of the newsvendor problem. The newsvendor problem is a classical model that is used to optimize the ordering quantity under uncertain demand. Due to its practical and theoretical importance, the newsvendor problem has been widely studied. Khouja (1999) presented a comprehensive review and classified the extensions of the newsvendor problem into eleven categories. Among those extensions are multi-product acquisition, newsvendor pricing, and supplier discounts.

Extensions to multi-product involve two or more products, usually with resource constraints. The constrained multi-product newsvendor model was first proposed by Hadley and Whitin (1963). Since ordering multiple products under budget or other constraints is common, the constrained multi-product newsvendor problem is widely studied in the last two decades. Representative work in this area includes that by Lau and Lau (1995, 1996), Erlebacher (2000), Abdel-Malek and Montanari (2005a, 2005b), and Niederhoff (2007). Incorporating pricing decision into the newsvendor problem was first presented by Whitin (1955), where selling price and stocking quantity are determined simultaneously. Then it was extensively studied by Petruzzi and Dada (1999), Webster and Weng (2008), and Chen and Bell (2009). Another important extension of the newsvendor problem is to take into account the supplier discount, which is a common policy for suppliers to promote their products. The notable work includes those of Pantumsinchai and Knowles (1991), Khouja (1996), Lin and Kroll (1997), and Zhang (2010).

So far, the three extensions to multi-product, pricing, and supplier discounts have been widely studied separately. To the best of our knowledge, it is the first investigation in the literature that studies these three issues in one integrated model. As discussed before, through the integrated model, the coordination

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of the up-stream and down-stream's decisions makes the problem more practical and challenging. Our objective is to develop the optimal acquisition and selling policy for the retailer, who faces uncertain demand and supplier discounts. Since suppliers provide quantity discounts, the product costs are piecewise linear. We develop a Mixed Integer Nonlinear Programming (MINLP) model to formulate the problem, and present a Lagrangian-based solution approach, which is very efficient for large-scale instances.

An outline of this paper is as follows: Section 2 provides a brief literature review of the related research. Section 3 presents the MINLP model for the problem. A Lagrangian-based solution approach is developed in Section 4, and numerical examples and computational results are presented in Section 5. We finally conclude the paper in Section 6.

2. Related research

There are numerous works that address the newsvendor problem and various extensions. Here we mainly review the related studies on the extensions to multi-product, quantity discount, and newsvendor pricing. For a more comprehensive review, the reader is referred to Khouja (1999). A brief comparison of the features of the reviewed papers with the proposed model in this research is illustrated in Table 1.

The multi-product acquisition under uncertain demand is usually modeled as the multi-product newsvendor problem. A budget or other resource constraints are always associated with the problem otherwise it can be treated as a single-product newsvendor problem. Hadley and Whitin (1963) first presented a formulation for the constrained multi-product newsvendor problem and developed a solution method for the problem. Then Lau and Lau (1995, 1996) presented a formulation and a solution procedure for the multi-product constrained newsvendor problem, which can efficiently solve large-scale problems involving 1000 products. Abdel-Malek and Montanari (2005a, 2005b) investigated the solution spaces for the multi-product newsvendor problem with one and two constraints, respectively. Abdel-Malek and Areeratchakul (2007) developed a quadratic programming model for the multi-product newsvendor problem with side constraints, which can be solved by familiar linear programming software packages such as Excel Solver and Lingo. Niederhoff (2007) presented an approximation method for the multi-product multi-constraint newsvendor problem by approximating the objective function with the piecewise linear interpolates. Zhang et al. (2009) presented a binary solution algorithm for the multi-product newsvendor problem with budget constraint. More articles on the multi-product newsvendor problem are included in Table 1.

Quantity discount is a common and effective policy for suppliers to promote their products. Quantity discount is based on the quantity of an item purchased—promoting the buyer to order large quantities of a given item. Pantumsinchai and Knowles (1991) formulated a single-period inventory problem with the consideration of standard container size discounts. Khouja (1995) formulated a newsvendor problem in which multiple discounts are used to sell excess inventory, while Khouja and Mehrez (1996) studied the multi-product constrained newsvendor problem under progressive multiple discounts. Khouja (1996) studied the newsvendor problem that considers both multiple discounts used by retailers to sell excess inventory and all-units quantity discounts offered by the suppliers. However, the model does not consider any resource constraint. Lin and Kroll (1997) investigated the single-item newsvendor problem with quantity discount and dual performance measure consideration. The solution approaches for the all unit quantity discount and incremental discount are developed. Zhang (2010) introduced supplier discounts to the constrained newsvendor problem, and presented a mixed integer nonlinear programming model. A Lagrangian heuristic is developed to solve the problem. However, the problem does not consider pricing decision.

By incorporating pricing into the newsvendor problem, Whitin (1955) first investigated the optimization problem of determining the stocking quantity and selling price simultaneously under uncertain demand environment. Petruzzi and Dada (1999) presented a comprehensive review and some meaningful

Table 1
Reviewed papers and their main contributions.

	Multi-product	Supplier discount	Newsvendor pricing	Main contribution or solution methods
Abdel-Malek et al. (2004)	Single-constraint			Generic iterative method (GIM)
Abdel-Malek and Montanari (2005a)	Single-constraint			Solution space analysis, extended GIM
Abdel-Malek and Montanari (2005b)	Two-constraint			Solution space analysis and Lagrangian method
Abdel-Malek and Areeratchakul (2007)	Multi-constraint			Quadratic programming approach
Chen and Chen (in press)	Single-constraint			Reservation considering and MCR algorithm
Erlebacher (2000)	Single-constraint			Heuristic approach
Lau and Lau (1995, 1996)	One- and multi-constraint			Solution procedure for large-scale problem
Moon and Silver (2000)	Single-constraint			Dynamic programming procedures
Niederhoff (2007)	Multi-constraint			Separable programming
Shao and Ji (2006)	Single-constraint			Fuzzy demand and hybrid intelligent algorithm
Vairaktarakis (2000)	Single-constraint			Dynamic programming
Zhou et al. (2008)	Multi-constraint			Return-CVaR consideration
Zhang et al. (2009)	Single-constraint			Binary solution method
Zhang (2010)	Multi-constraint	All-unit		Lagrangian method
Khouja (1996)		All-unit	Multi-discount	Analytical method
Lin and Kroll (1997)		All-unit and incremental		Analytical method
Pantumsinchai and Knowles (1991)		Standard container size		Dynamic programming
Chen and Bell (2009)			Linear	Return consideration and analytical approach
Karakul (2008)			Linear	Clearance market considering
Khouja (1995)			Multi-discount	Analytical approach
Khouja and Mehrez (1996)	Single-constraint		Multi-discount	Lagrangian method
Serel (2008)			Linear	Risky supply consideration
Webster and Weng (2008)			Linear	Supply chain environment
Petruzzi and Dada (1999)			Linear and nonlinear	Review and meaningful extensions
Pan et al. (2009)			Linear	Two selling period
This paper	Single-constraint	All-unit	Linear	Combine all and Lagrangian method