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Incorporating quantity discounts to the EOQ model with transportation costs

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Abstract

This article extends a previous Economic Order Quantity (EOQ) model with two modes of transportation, namely truckload (TL) and less than truckload (LTL) carriers, by introducing all-units and incremental quantity discount structures into the analysis. Exact algorithms are developed for each quantity discount structure to compute optimal policies for single-stage models over an infinite planning horizon.

Keywords: Inventory; Quantity discounts; Transportation costs

1. Introduction

This article incorporates quantity discounts on the purchased units into the Economic Order Quantity (EOQ) model with transportation costs. Some of the key components of transportation that companies must examine are the various modes of transportation available and their costs. Different modes of shipping freight are typically categorized as either truckload (TL) transportation or less than truckload (LTL) transportation. In TL transportation, there is a fixed cost per load up to a given capacity. With TL transportation, a company may use less than the capacity available and transport this freight at the cost of a full load. This fixed cost comprises the costs due to the truck, driver, and operating expenses. Examples of TL transportation are the usage of trucks, railway wagons, and overseas containers as ways of shipping freight; however, in some cases, the quantity of freight may not be large enough to justify the cost associated with a TL shipment. For these small quantities, a LTL carrier may be used. In LTL, the unit transportation cost is constant. An example of LTL transportation is a company that uses a third-party carrier such as United Parcel Service to ship freight.

Regarding quantity discounts on the purchased units, these are given to customers in order to encourage them to place larger orders. In this article, we consider both all-units and incremental quantity discounts. An all-units quantity discount is a discount given on every unit that is purchased after the purchasing quantity exceeds a given level (breakpoint). In an incremental quantity discount,

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the discount only applies to units that are purchased beyond the level at which the discount is given. In this article, a previous EOQ model with two modes of transportation, namely TL and LTL carriers, is extended by introducing all-units and incremental quantity discount structures into the analysis. Exact algorithms are developed for each quantity discount structure to compute optimal policies for single-stage inventory models over an infinite planning horizon. These algorithms are particularly useful in practical situations where high setup costs may occur. For instance, a current trend in the consumer products' industry is that, to remain competitive, companies are continually searching to make cosmetic changes to their products each time they are re-ordered (Hitachi Consulting, 2005). This is performed to reduce the high rates of return on dated products, or simply to evolve and improve value of current products (e.g. new packaging, Gordon, 2005; Cecere and Hofman, 2007). So, as one might imagine, companies are constantly incurring redesign costs as part of their products' life management to try to bring innovative changes to the market more rapidly. All these costs are usually considered as part of the setup cost of a product. Moreover, some of these products are outsourced to countries, such as China or India. Thus, order quantities are usually large, not only due to the cost of redesign but also due to the potential cost savings in transportation.

The remainder of this article is organized as follows. In Section 2, previous work related to this research is summarized. In Section 3, the notation and assumptions used throughout this article are stated and briefly explained. In addition, the development of the models with all-units and incremental quantity discounts, and the corresponding algorithms to find the optimal reorder interval are presented in Sections 3.3 and 3.4, respectively. Each algorithm is illustrated with a numerical example. Finally, some conclusions and future research issues are summarized in Section 4.

2. Literature review

The EOQ formula developed by Harris (1990) is the most fundamental of all inventory models. It describes the important trade-off between fixed ordering costs and holding costs, given that the demand rate is constant, inventory shortages are not allowed, and replenishments are instantaneous. Well-known extensions of the basic EOQ model consider inclusion of shortage costs, extension to the case of a finite production rate, and consideration of quantity discounts. Chopra and Meindl (2007), Johnson and Montgomery (1974), and Nahmias (2001) include sections that offer complete analyses on these extensions.

Regarding *inventory models with quantity discounts*, most research work has focused on all-units and incremental quantity discounts. An overview of the quantity discounts research is presented by Benton and Park (1996). Munson and Rosenblatt (1998) presented an exploratory study of 39 companies and their different discount strategies in practice. They found that 95% of the companies they studied either offer or receive some type of all-units quantity discounts. In addition, 37% of these firms offer or receive incremental quantity discounts. Hu and Munson (2002) presented a heuristic for incremental quantity discounts with constant demand over a finite horizon. Hu et al. (2004) suggested a modification of the classical Silver-Meal heuristic under the incremental quantity discount case to improve the results presented by Hu and Munson (2002).

Analyses on multi-stage systems with quantity discounts have also been performed. For example, Chen et al. (2001) included order frequency as part of the discount function. Specifically, this article studied a distribution channel where a supplier distributes a single product to retailers, who, in turn, sell the product to customers. Order quantity and annual sales volume were other components of their discount. Anupindi and Bassok (1998) also considered volume-based discounts. Recently, Munson and Rosenblatt (2001) considered a decentralized quantity discount model with three levels: a supplier, a manufacturer, and a retailer. They explored the benefits of using quantity discounts in order to decrease costs.

Concerning *inventory models with transportation costs*, the two different modes of transportation analyzed in this article have been extensively explored by different authors. Inventory models with TL transportation have been studied by Aucamp (1982), Iwaniec (1979), and Lippman (1969, 1971). In their models, the cost per load does not change with the number of truckloads. Lee (1986), in contrast, considered the classical EOQ model with setup cost including a fixed cost and freight cost, where the freight cost is subject to quantity discounts. Lee (1989) also developed a dynamic programming algorithm for TL shipments