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An economic order quantity with imperfect quality and quantity discounts

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ABSTRACT

Previous studies in the issue of inventory models with imperfect quality assumed the defectives could be sold in a batch by the end of the inspection process and the manufacturing systems were push systems. However, the above assumptions may not be true in the pull system in which buyer is powerful. Therefore, in this paper, we develop a new inventory model for items with imperfect quality and quantity discounts where buyer has exerted power over its supplier. Based on the concept of powerful buyer, there are three considerations included in this new model: (1) the order quantity is manufactured at one setup and is shipped over multiple deliveries, (2) the defectives are screened out by a 100% inspection for each shipment but sold in a batch by the end of inspection at the last shipment of each cycle, and (3) the supplier offers quantity discounts to response the request of the powerful buyer. Further, an algorithm is developed to help the powerful buyer to determine the optimal order policy accurately and quickly. Two numerical examples are available in this paper to illustrate the proposed model and algorithm. Besides, based on the numerical examples, a sensitivity analysis is made to investigate the effects of four important parameters (the inspection rate, the defective rate, the receiving cost, and the ordering cost) on the optimal solution.

1. Introduction

In the last few decades, both academics and practitioners have shown that effective inventory management for company is one of the key factors for success in challenging business environment. There are several differences, which can be considered to reflect variations in the structure of the inventory system, between existing systems (Hadley and Whitin [1]). Generally, the operating doctrines or decision rules which guide a company's attempts to maximize profit while satisfying market demands depend upon sound inventory policies and the utilization of models (Burwell et al. [2]). The first and the simplest model to explore inventory topic is economic order quantity (EOQ) inventory model. Ever since that model was developed in the earliest decades of this century, many excellent extensions were made due to unrealistic assumptions. For example, with respect to the frequent deliveries, Pan and Liao [3], Larson [4], and Ramasesh [5] have developed EOQ-based models to discuss the effect of lot-splitting shipments on total costs. With respect to the quantity discounts, Benton and Park [6], Li and Liu [7], and Munson and Hu [8] have explored the theory of inventory systems for both deterministic and stochastic demand and have illustrated the economic advantages from quantity discounts. Recently, a new economic order quantity (EOQ) model for items with imperfect quality has received considerable attention by many researches because, in practice, ordered items may exist defectives.

The first proponent focusing on the issue of EOQ model with imperfect quality properly was Porteus [9] in which he developed a simple model to capture a significant relationship between quality and lot size. Rosenblatt and Lee [10] explored

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economic production cycles with imperfect production process, in which the defective items could be reworked instantaneously at a cost, and found that the presence of defective products motivates smaller lot sizes. Lee and Rosenblatt [11] dealt with the possibility of process deterioration and the existence of defective items in the production lot based on economic manufacturing quantity (EMQ) model. Unlike the above mentioned works, Salameh and Jaber [12] assumed each order contains a random fraction of imperfect quality items, in which the defective items could be sold in a single batch by the end of the 100% screening process, with a known probability. Thereafter, many extensions based on Salameh and Jaber's [12] work were developed. Cardenas-Barron [13] corrected an error in Salameh and Jaber but did not affect the main idea. Goyal and Cardenas-Barron [14] developed a simple approach which can obtain a near-optimal approximation for the order quantity. Papachristos and Konstantaras [15] discussed many of its assumptions, most notably those focusing on avoiding shortages. More recently, Maddah and Jaber [16] identified a flaw in the Salameh and Jaber's [12] work and developed a new model that rectified the flaw using renewal theory. They also extended the analysis by allowing for several batches of imperfect quality items to be consolidated and shipped in a single lot. Building upon the work of Salameh and Jaber, Huang [29] extended their idea into the optimal integrated vendor–buyer inventory policy for flawed items. In the same time, Chang [18] employed fuzzy set theory to this topic. Chan et al. [19] developed an economic production model in which products were classified as good quality, good quality after reworking, imperfect quality and scrap. Recently, Rezaei [20] and Wee et al. [21] extended the traditional EPQ/EOQ model by accounting for imperfect quality items when using the backorder EPQ/EOQ formulae. However, a flaw exists in Wee et al.'s [21] model that they neglected the backorder must be eliminated in the beginning of replenishment period with perfect items. Fortunately, Eroglu and Ozdemir [22] avoided this flaw and developed a model by allowing shortages and scrap items. More recently, Chung et al. [23] incorporated concepts of the basic two warehouses to generalize the EOQ model with imperfect quality. Note that the works of Eroglu and Ozdemir [22] is based on the assumption of $p \leqslant 1 - D/x$ suggested by Salameh and Jaber [12]. However, Papachristos and Konstantaras [15] pointed this condition may not avoid the occurrence of shortage, but failed to give correction to this flaw. Jaber et al. [30] have found the inspection rate is relatively much higher than the demand rate, and with learning effects the percentage defectives per shipment reduces to small value. Lin [24] further suggested if the defective rate is with boundary (e.g. uniform distribution) then shortages can be avoid. Given the attention received by the Salameh and Jaber's model, however, we feel their work may be modified under the case of powerful buyer who has exerted power over its supplier.

Note that all of the previous research in the field of inventory models with imperfect quality implicitly assumed that the manufacturing systems are push systems. Actually, modern manufacturing are pull systems, in which the trigger is in the buyer's hand. In pull systems, delivers must be made on an as-needed basis only (Nahmias [25]). Kim and Ha [17] indicated that small lot sizing improves the systems productivity by obtaining lower levels of inventory and scrap, lower inspections for incoming parts, and earlier detection of defects, etc., even though possible higher delivery costs and loss of discount rates may be incurred. However, these weaknesses mentioned by Kim and Ha [17] may not be true when buyer has exerted power over its supplier. Many suppliers are reported feeling squeezed and pressured by powerful buyers into taking expensive actions such as lower prices, accelerating delivery times and offering special allowances (Bloom and Perry [26]). Based on the above argument, this paper incorporates the views of Salameh and Jaber [12] and Bloom and Perry [26] into the case with lot-splitting shipments and quantity discounts. Specifically, this paper deals with the powerful buyer's inventory model for items with imperfect quality and quantity discounts in which the buyer's order quantity is manufactured at one setup and shipped in equal amounts over multiple delivers. Also, the supplier undertakes the delivery costs itself but the buyer pays the receiving cost for each shipment. Furthermore, the defectives are screened out by a 100% inspection for each shipment but sold in a batch by the end of inspection at the last shipment of each cycle.

2. Notation and assumptions

The following notation and assumptions are used hereinafter to develop the proposed model.

Notation

q	order size
D	the demand rate
x	the screen rate, $x > D$
p	the defective percentage in q
$f(p)$	the probability density function of p
m	the selling price per unit
v	the salvage value of per defective item, $v < c_j$
d	the screening cost per unit
I	the percentage of unit price
K	the ordering cost per order
N	the number of shipments per cycle (integer value)
R	the receiving cost per shipment
T	planning horizon
q_s	the size of shipment for each delivery which is given by $q_s = q/N$
t_s	the inventory depletion time for each shipment