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Jointly pricing and ordering for a multi-product multi-constraint newsvendor problem with supplier quantity discounts

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ABSTRACT

We present an extension to the multi-product newsvendor problem by incorporating the retailer's pricing decision as well as considering supplier quantity discount. The objective is to maximize the expected profit of the retailer through jointly determining the ordering quantities and selling prices for the products, subject to multiple capacity constraints. We formulate the problem as a Generalized Disjunctive Programming (GDP) model and develop a Lagrangian heuristic approach for its solution. Randomly produced instances involving up to 1000 products are used to test the proposed approach. Computational results show that the Lagrangian heuristic approach can present very good solutions to all instances in reasonable time.

1. Introduction

In classic newsvendor problem (NP), the retailer has to determine the optimal order quantity to obtain a balance between the costs of shortages and leftovers, while the probability distribution of the demand is assumed to be known. As its theoretic and practical meaning, it has been widely studied. Khouja [1] presents a comprehensive review and classified its extensions into 11 categories, such as extensions to different objectives and utility functions, extensions to different supplier pricing policies, extensions to random yields, and extensions to constrained multi-product. Some suggestions for the future research are proposed, one of which is to combine two or more of these extensions to make the newsvendor problem more practical.

In practical business, retailers usually have to jointly make two decisions: determining the acquisition quantities for their products and competitive prices to sell them out. Both these decisions have to be made under highly uncertain demand environment. Furthermore, the retailers need to consider the quantity discounts provided by their suppliers. Quantity discounts have been widely employed by suppliers to promote their products. It is attractive to the retailers, because it provides a potential way to reduce the unit acquisition costs of the products. But considering quantity discount also makes the decision process more complex.

Motivated by the challenges of practical business and theoretic requirements, three extensions to the newsvendor problem, constrained multi-product, newsvendor pricing and supplier quantity discount, are combined in one model to optimize the newsvendor's acquisition and pricing policy. Moreover, Generalized Disjunctive Programming (GDP) techniques are

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introduced to formulate the problem, instead of Mixed Integer Programming (MIP) which usually is employed in traditional literatures. We also consider the multiple capacity constraints of the retailer and develop a Lagrangian heuristic approach to obtain near optimal solution for the problem.

Since the constrained multi-product newsvendor problem (MPNP) is presented by Hadley and Whitin [2], it has attracted many researchers' attention. Vairaktarakis [3] considers the robust version of MPNP with single budget constraint and presents three minimax regret formulations. Shao and Ji [4] present three types of formulations for MPNP with fuzzy demands and budget constraint: expected profit model, the most profit model and α-profit model. Abdel-Malek and Areeratchakul [5] develop a quadratic programming solution model for MPNP with side constraints. Niederhoff [6] presents an approximating programming technique for MPNP. Abdel-Malek et al. [7] present a formulation for MPNP with random yield. Abdel-Malek and Montanari [8] present an analysis on the solution space of MPNP with one capacity constraint, while Abdel-Malek and Montanari [9] analyze the solution space of MPNP with two constraints. Moreover, many solution approaches, such as heuristic algorithm [10], dynamic programming procedure [11], Generic Iterative Model [12], and binary solution algorithm [13], have been developed to solve the problem. Although MPNP has been widely investigated, most proposed formulation techniques and solution approaches are used to solve problems involving about 10 products. Lau and Lau [14,15] are the few literatures that present solution approaches for problem involving thousand products.

By incorporating pricing decision into the newsvendor problem, Whitin [16] first investigates the optimization problem of determining the stocking quantity and selling price simultaneously under uncertain demand environment. Petruzzi and Dada [17] present a comprehensive review and some meaningful extensions for the newsvendor pricing problem. Petruzzi and Dada [18] further analyze the jointly determining inventory and pricing decisions for the newsvendor problem in a two-period/two-market retail setting. Karakul [19] studies the joint pricing and procurement of fashion products in the existence of clearance markets. Granot [20] analyzes the effect of price and order postponement in a decentralized newsvendor model with multiplicative and price-dependent demand. Chen and Bell [21] address the simultaneous determination of price and inventory replenishment when customers return product to the firm. Pan et al. [22] construct a two-period model to determine pricing and ordering policy for a dominant retailer with demand uncertainty in a declining price environment. Furthermore, He et al. [23] consider an extension for the newsvendor problem where the demand is assume to be uncertain and sensitive to both retail price and effort. As the newsvendor pricing problem can coordinate the acquisition and market demand under uncertain environment, it has received increasing attention in recent years. But most current literatures investigate its application in different supply chain environment and gain managerial insights, few of them study the optimization problem for multi-product case.

Quantity discount is a common and effective policy for suppliers to promote their products. If quantity discount is provided by suppliers, the retailers can procure products at a lower unit price when the ordering quantity is over a certain value – the price break point. Khouja [24] investigates the newsvendor problem which considers both multiple discounts used by retailers to sell excess inventory and all-units quantity discounts offered by the suppliers. Lin and Kroll [25] study the single-item newsvendor problem with quantity discount and dual performance measure consideration. The solution approaches for the all unit quantity discount and incremental discount are developed. Besides, supplier quantity discount is also widely considered in supplier selection, and a relative review can be found in Zhang and Ma [26].

Shi and Zhang [27] incorporate supplier discounts and newsvendor pricing into the multi-product newsvendor problem with a budget constraint. The problem is formulated as MINLP models. When supplier discounts are integrated, mixed integer programming is the most commonly adopted technique. In this paper, we introduce a new technique – Generalized Disjunctive Programming (GDP) to formulate the multi-product newsvendor pricing problem with supplier quantity discount, and compare it with the MINLP model.

Until now, newsvendor problem and its extensions have regained considerable attention in both practice and academia, and it is viewed as a better strategy for the highly uncertain business environment. As far as we know, there are few literatures that consider the three extensions, constrained multi-product, newsvendor pricing and supplier discounts, in one optimization model. Our objective is to develop the optimal acquisition and selling policy for the multi-product newsvendors under multiple capacity constraints.

The organization of the paper is as follows. Section 2 presents a GDP formulation for the problem. A solution approach based on Lagrangian relaxation is developed in Section 3, and computational results are reported in Section 4. We finally conclude the paper in Section 5.

2. Model formulation

In this paper, all unit quantity discount is provided by the supplier. The demands for all products are assumed to be linear price-dependent and additive stochastic, that is, $\widetilde{D}_i(p_i,u_i) = D_i(p_i) + u_i$, where $D_i(p_i) = a_i - b_i p_i$ is the mean demand of product i as a function of price and u_i is a random variable defined on the range $[A_i,B_i]$ with a known distribution and $E[u_i] = 0$. This demand form is usually adopted whenever the variance of demand is unaffected by the deterministic demand level, and it has been widely applied in operations research literatures, such as Petruzzi and Dada [17], Karakul [19], Pan et al. [22].