PERGAMON

Computers & Industrial Engineering 38 (2000) 67–77

computers & industrial engineering

www.elsevier.com/locate/dsw

Fixed cycle ordering policies for capacitated multiple item inventory systems with quantity discounts

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Abstract

We consider the problem of stocking inventories of multiple items that share a common resource, like a warehouse space, where the vendor of the items offers quantity discounts. For the classic version of this problem without quantity discounts, it is well known that a fixed cycle approach, where the time between order replenishments for all items is the same, is a cost effective heuristic solution methodology, one that in some instances is preferred to both independent cycle and non-stationary approaches. In this paper, we demonstrate that a fixed cycle approach can also be used to heuristically solve the quantity discount case. Computational results suggest that this approach is often more cost effective than competing solution methodologies, particularly when the resource constraint is tight.

Keywords: Inventory; Heuristics; Quantity discounts; Resource constraint

1. Introduction

Purchasing multiple items from a vendor is a standard business practice. It is also common for the buyer to have a resource constraint, typically arising from a budgetary limitation or warehouse capacity restriction (Pirkul & Aras, 1985). As a result, the problem of making ordering decisions for capacitated multiple item inventory systems is regarded as an important topic of inventory management (Rosenblatt & Rothblum, 1990), and is discussed in many texts on the subject, including the classic treatise by Hadley and Whitin (1963).

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PII: S0360-8352(00)00029-2

In the fundamental version of this problem, demand rates are constant, replenishment of stock is instantaneous upon delivery, shortages are not allowed, the unit prices charged by the supplier for the items are independent of the order quantities, and there is a single resource constraint that limits maximal inventory levels over time. These simplifying assumptions are made to achieve a tractable mathematical model. This problem is referred to as the undiscounted problem. In this paper, we consider a generalization of the undiscounted problem, called the discounted problem, where the supplier of the items offers quantity discounts. Quantity discount offerings represent another standard business custom (Muson & Rosenblatt, 1995). Typically, these discounts are either all-units or incremental (Tersine, 1988; Widrick, 1985).

2. Literature review

Work on solving the discounted problem has involved extending ideas developed for solving the undiscounted problem. Three heuristic solution approaches exist for the undiscounted problem. One assumes independent cycle times for each of the various items, and uses Lagrangian relaxation to guarantee that the total available capacity is not exceeded at the eventual points in time when all orders reach a simultaneous peak (Hadley & Whitin, 1963). The second approach uses the same fixed order cycle for all items, and performs phasing within the cycle to stop all orders from arriving at once. Intuitively, these order timings can make the capacity constraint less restrictive and, hence, less expensive (Page & Paul, 1976). The third approach relaxes an implicit assumption of both the independent cycle and the fixed cycle approaches that all order quantities are stationary or invariant over time, and thereby offers potential cost savings over the other two approaches (Güder, Zydiak & Chaudhry., 1995).

As shown in (Rosenblatt, 1981), there are cases where the independent cycle and the fixed cycle approaches are each preferred. Simulation results by Page and Paul (1976) suggest that as the resource constraint gets tighter, the fixed cycle approach shows greater improvement over the independent cycle approach. In addition, the non-stationary approach is shown in Güder et al. (1995) to always be preferred to the independent cycle approach, and in some cases, preferred to the fixed cycle approach. Thus, the fixed cycle and non-stationary methods appear to be the dominant approaches in solving the undiscounted problem.

For the discounted problem with either all-units or incremental price breaks, there are two families of heuristic solution approaches reported in the literature that parallel those for the undiscounted problem. One assumes independent cycle times and uses Lagrangian relaxation (Pirkul & Aras, 1985; Güder et al., 1994; Rubin and Benton, 1993). The other uses a non-stationary approach, and is shown in Güder and Zydiak. (1997) to outperform stationary independent cycle approaches. Note, however, that applications of a fixed cycle approach to the discounted case have never been reported in the literature, to the best of our knowledge. This paper details the first such application,and shows that significant cost savings over the rival approaches are possible. As it has been demonstrated in the literature (Güder et al., 1994) that approaches assuming all-units quantity discounts can easily be adapted to address incremental discounts, we assume, without loss of generality (wlog), that all-units discounts are being offered.