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An inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging

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Abstract

Wee (International Journal of Production Economics 59 (1999) 511), in his interesting paper presented a deterministic inventory model with the following characteristics. Quantity discount schemes for the unit cost, partial backlogging at a fixed rate, deterioration of stock in time and demand rate being a linear function of the selling price. In this article we generalize the work of Wee (1999). More specifically, we consider a model where the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent function, which ensures that the rate of backlogged demand increases as the waiting time to the following replenishment point decreases.

Keywords: Deterioration; Inventory; Pricing; Quantity discount; Partial backlogging

1. Introduction

The deterioration of inventory in stock during the storage period constitutes an important factor, which has attracted the attention of researchers. Deterioration, in general, may be considered as the result of various effects on the stock, some of which are damage, spoilage, obsolescence, decay, decreasing usefulness, and many more. The first inventory model where the factor of deterioration was considered as significant seems to be that proposed by Ghare and Shrader (1963). These authors studied a model, having a constant rate of deterioration and a constant rate of demand over a finite planning horizon. The model of Ghare and Schrader was extended by Covert and Philip (1973) by introducing variable rate of deterioration. A further generalization to the above models was proposed by Shah (1977) by considering a model allowing complete backlogging of the unsatisfied demand.

There is an extended literature concerning deteriorating inventories. A common characteristic to the most of these models is that they allow shortages, while the unsatisfied demand is completely backlogging. The consideration of partial backlogging for non-perishable product has been undertaken by Montgomery et al. (1973), Rosenberg (1979) and others. The study of models allowing for partial backlogging of unsatisfied demand and deterioration of inventory in stock has been pursued by Wee (1995), Abad (1996), and Chang and Dye (1999).

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All these models, and many more, treated the demand rate as constant or time-dependent function. However, in the real-world application the unit selling price is effected the demand, as low selling price exposures demand, while high selling prices decline demand records to zero. This reality has led researchers to consider and study models where demand is taken as a function of the selling price, the so-called optimal pricing policies.

Eilon and Mallaya (1996), were the first authors who studied the pricing policy. They considered perishable items with maximum shelf life and no deterioration before the expiration date. Deterioration is a rather continuously going on phenomenon affected inventory stock. The model by Cohen (1977), takes into account this pattern of deterioration by introducing an exponentially decaying scheme for the inventory in stock and treats demand rate as a linear function of selling price. The simultaneous optimal pricing and ordering levels are determined. Research continues with Kang and Kim (1983), Aggarwall and Jaggi (1989), Abad (1988). Shiue (1990) developed an inventory model over a prescribed scheduling period, taking general rate of deterioration, constant demand rate, partial backlogging. Further, he used three purchasing price, scheme i.e. all units quantity discount, incremental quantity discount and time discount. In a most recent article, Wee (1999) studied an inventory model considering, joint pricing and replenishment policy, where stock deteriorates with time following the Weibull rate of deterioration. The purchase cost for the product follows the all units quantity discount scheme and demand rate is a decreasing linear function of the selling price. The model allows for shortages, which are partially backlogged at a constant rate.

In this article we generalize the work of Wee (1999) as follows. The demand rate is described by any convex decreasing function of the selling price and instead of a constant rate of partial backlogging we consider a variable backlogging rate, as proposed by Abad (1996). For this model we found the optimal solution and we compare it, via examples, to the approximated results produced by Wee (1999). Moreover, we propose a correction for the revenue function considered in the model of Wee.

2. Notation and assumptions

The following notation is used throughout the paper:

T	cycle length
T_1	inventory cycle interval with positive stock (decision variable)
q	order quantity (units/cycle)
$I(t)$	the inventory level at time t
I_m	maximum starting inventory level of the cycle (units)
I_b	the per cycle amount of shortages backlogged (units)
I_l	the per cycle lost sales quantity (units)
s	unit selling price (decision variable)
$d(s)$	deterministic demand rate for the product, a function of the selling price s (units/unit time)
m_i	the ith price breaking point, $i = 1, \ldots, n$
$u_i = u_i(q)$	per unit material cost
c_1	fixed cost per order
c_2	per unit holding cost per unit time $= c_h + Iu_i(q)$, where c_h stands for the holding cost that is independent of the unit cost $u_i(q)$, and I is the fixed annual holding cost fraction of unit cost $u_i(q)$
c_3	shortage cost per unit backordered per unit of time
c_4	lost sales cost per unit of lost sale

The inventory model is developed under the following assumptions:

1. Demand rate $d(s)$ is any non-negative, continuous, convex, decreasing function of selling price, defined in the interval $[0, s_{\max}]$ and taking values on the interval $[0, d_{\max}]$, such that $\partial^2(sd(s))/\partial s^2 \leqslant 0$ for all s belonging to $[0, s_{\max}]$ going to zero as s goes to $s_{\max}$. Note that the selling price s is a decision variable. Its value has to be decided at the beginning of the cycle and remains constant over it.
2. The replenishment rate is infinite.
3. The lead time is zero.
4. The time to deterioration of the item is distributed as Weibull (α, β), that is, at time t, $\alpha\beta t^{\beta-1} I(t)$ units have been deteriorated, where α, β are positive parameters.