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Decision Support Systems 37 (2004) 485–500

Decision Support Systems

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Arbitrage pricing theory-based Gaussian temporal factor analysis for adaptive portfolio management

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Available online 17 July 2003

Abstract

Ever since the inception of Markowitz's modern portfolio theory, static portfolio optimization techniques were gradually phased out by dynamic portfolio management due to the growth of popularity in automated trading. In view of the intensive computational needs, it is common to use machine learning approaches on Sharpe ratio maximization for implementing dynamic portfolio optimization. In the literature, return-based approaches which directly used security prices or returns to control portfolio weights were often used. Inspired by the arbitrage pricing theory (APT), some other efforts concentrate on indirect modelling using hidden factors. On the other hand, with regard to the proper risk measure in the Sharpe ratio, downside risk was considered a better substitute for variance. In this paper, we investigate how the Gaussian temporal factor analysis (TFA) technique can be used for portfolio optimization. Since TFA is based on the classical APT model and has the benefit of removing rotation indeterminacy via temporal modelling, using TFA for portfolio management allows portfolio weights to be indirectly controlled by several hidden factors. Moreover, we extend the approach to some other variants tailored for investors according to their investment objectives and degree of risk tolerance.

Keywords: Temporal factor analysis; Arbitrage pricing theory; Portfolio optimization; Sharpe ratio; Downside risk; Upside volatility

1. Introduction

Portfolio management has evolved as a core decision-making activity for investors and practitioners in the financial market nowadays. Prior to the inception of Markowitz's modern portfolio theory [11], theoretical research on investments has concentrated on modelling expected returns [2].

During the early stage of its development, portfolio optimization was often constrained by its static implementation. Unlike dynamic portfolio optimization by which the optimal portfolio weights were tracked over time based on updated market information, the weights determined using static optimization techniques could not adapt to market changes within the investment horizon.

Despite dynamic portfolio optimization being powerful, it turned out to be a problem that required intensive computation. Recall that the most natural technique for solving dynamic portfolio optimization problems was stochastic dynamic programming. However, this approach was often compromised by several factors such as the curse of dimensionality when too many state variables were involved [7]. In

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doi:10.1016/S0167-9236(03)00082-4

general, practical considerations such as taxes and transactions costs also increased the number of state variables in the objective function.

In fact, this problem could be better solved via some popular machine learning approaches [3,12,13,21] which required the optimal parameters to be adaptively learned over time, and consequently, we have the term adaptive portfolio management. Among the various methodologies suggested, the most popular one is based on maximizing the well-known Sharpe ratio [17]. In implementation, trading could be based on training a trading system on labelled data [12] or directly maximizing the expected profit via the so-called adaptive supervised learning decision networks [8,21]. In this paper, these approaches were generally referred to as return-based portfolio management because they either explicitly treated the weights as constants or depend directly on the security price or returns.

Inspired by the arbitrage pricing theory (APT) in finance, which assumes that the cross-sectional expected returns of securities is linearly related to k hidden economic factors, typical statistical techniques such as principal component analysis (PCA), independent component analysis (ICA) [1,22] and maximum likelihood factor analysis [10] have been used. However, should we adopt either PCA or ICA for estimating the hidden factors, we have to compromise on the terms of zero noise. Likewise, we have to make a compromise on rotation indeterminacy if we use conventional factor analytic techniques.

In fact, many researchers also realized that variance was not appropriate for quantifying risk in the Sharpe ratio because it counted positive returns as risk. For instance, Fishburn used the lower partial moment (LPM) [5] of returns called downside risk to replace the traditional variance measure. Moreover, similar ideas were adopted for implementing portfolios optimization [8,9].

In this paper, we aim to investigate using the technique temporal factor analysis (TFA) [18] for portfolio optimization. Since TFA is based on the classical APT model and has the benefit of removing rotation indeterminacy via temporal modelling, using TFA for portfolio management allows portfolio weights to be indirectly controlled by several hidden factors. Moreover, we can extend the approach to some other variants tailored for investors according to their risk and return objectives.

The rest of the paper is organized in the following way. Sections 2 and 3 briefly review the APT and the Gaussian TFA models, respectively. Section 4 illustrates how the APT-based adaptive portfolio management can be effected with algorithms proposed in this paper. Three variants of the APT-based Sharpe ratio maximization technique are studied in Section 5. Section 6 concludes the paper.

2. Review on arbitrage pricing theory

The APT begins with the assumption that the $n \times 1$ vector of asset returns, R_t, is generated by a linear stochastic process with k factors [14–16]:

$$R_t = \bar{R} + Af_t + e_t \tag{1}$$

where f_t is the $k \times 1$ vector of realizations of k common factors, A is the $n \times k$ matrix of factor weights or loadings and e_t is an $n \times 1$ vector of asset-specific risks. It is assumed that f_t and e_t have zero expected values so that $\bar{R}$ is the $n \times 1$ vector of mean returns. The model addresses how expected returns behave in a market with no arbitrage opportunities and predicts that an asset's expected return is linearly related to the factor loadings or

$$\bar{R} = R_f + Ap \tag{2}$$

where R_f is an $n \times 1$ vector of constants representing the risk-free return, and p is $k \times 1$ vector of risk premiums. Similar to the derivation of CAPM, Eq. (2) is based on the rationale that unsystematic risk is diversifiable and therefore should have a zero price in the market with no arbitrage opportunities.

3. Overview of temporal factor analysis

Suppose the relationship between a state $y_t \in \mathbb{R}^k$ and an observation $x_t \in \mathbb{R}^d$ is described by the first-order state-space equations as follows [18,19]:

$$y_t = By_{t-1} + \varepsilon_t, \tag{3}$$

$$x_t = Ay_t + e_t, \qquad t = 1, 2, \ldots, N \tag{4}$$

where ε_t and e_t are mutually independent zero-mean white noises with $E(\varepsilon_i \varepsilon_j^T) = \Sigma_\varepsilon \delta_{ij}$, $E(e_i e_j^T) = \Sigma_e \delta_{ij}$,