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Numerical methods for optimal dividend payment and investment strategies of regime-switching jump diffusion models with capital injections[☆]

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ABSTRACT

This work focuses on numerical methods for finding optimal investment, dividend payment, and capital injection policies to maximize the present value of the difference between the cumulative dividend payment and the possible capital injections. The surplus is modeled by a regime-switching jump diffusion process subject to both regular and singular controls. Using the dynamic programming principle, the value function is a solution of the coupled system of nonlinear integro-differential quasi-variational inequalities. In this paper, the state constraint of the impulsive control gives rise to a capital injection region with free boundary, which makes the problem even more difficult to analyze. Together with the regular control and regime-switching, the closed-form solutions are virtually impossible to obtain. We use Markov chain approximation techniques to construct a discrete-time controlled Markov chain to approximate the value function and optimal controls. Convergence of the approximation algorithms is proved. Examples are presented to illustrate the applicability of the numerical methods.

1. Introduction

Designing dividend payment policies has long been an important issue in finance and actuarial sciences. Because of the nature of their products, insurers tend to accumulate relatively large amounts of cash, cash equivalents, and investments in order to pay future claims and avoid insolvency. The payment of dividends to shareholders may reduce an insurer's ability to survive adverse investment and underwriting experience. A practitioner will manage the reserve and dividend payment against asset risks so that the company can satisfy its minimum capital requirement.

Stochastic optimal control problems on dividend strategies for an insurance corporation have drawn increasing attention since the introduction of the optimal dividend payment model proposed by De Finetti (1957). There have been increasing efforts on using advanced methods from the toolbox of stochastic control to study the optimal dividend policy; see Asmussen and Taksar (1997), Gerber and Shiu (2004), Jin, Yin, and Zhu (2012), and Yin, Jin, and Yang (2010). Browne studied the optimal investment strategy for a firm with the constraint of probability of ruin in Browne (1995). A comprehensive study of switching diffusions with state-dependent switching is in Yin and Zhu (2010). Azcue and Muler (2010) analyzed the problem of the maximization of total discounted dividend payment for an insurance company. Empirical studies indicate, in particular, that traditional surplus models fail to capture more extreme movements such as market switching. To reflect reality, much effort has been devoted to produce better models. One of the recent trends is to use regime-switching models. Hamilton (1989) introduced a regime-switching time series model. Recent work on risk models and related issues can be found in Yang and Yin (2004). Optimal dividend strategies were studied in a regime-switching diffusion model in Sotomayor and Cadenillas (2011).

To maximize the expected total discounted dividend payments, the company will bankrupt almost surely if the dividend payment is paid out as a barrier strategy. In practice, Dickson and Waters

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(2004) suggested that capital injections can be taken into account to avoid insolvency when capital reserve is insufficient. Furthermore, transaction cost will be considered; see also Sethi and Taksar (2002), Kulenko and Schimidli (2008), and Yao, Yang, and Wang (2011). Whenever the company is on the verge of financial ruin, the company has the opportunity to raise sufficient funds to survive. A natural payoff function is maximizing the difference between the expected total discounted dividend payment and the capital injections with costs until bankruptcy under the optimal controls.

In this work, we aim to obtain the optimal dividend payment and investment strategies using the collective risk model under the Markovian regime-switching setting with capital injections. We allow the investment of surplus in a continuous-time financial market and the management of the dividend payment policy. In our model, borrowing money to do risky investment is not allowed. The insurers cannot put too much money in risky assets for the sake of risk management. That is, there is a natural constraint on the portfolio so that the total weight of the risky assets should be no more than 1. Another constraint on the investment is that short selling risky asset is prohibited. Hence, the proportion of capital invested in the risky asset is denoted as a regular control $u \in [0, 1]$. In addition, a dividend process is not necessarily absolutely continuous. In fact, dividends are usually paid out at random discrete times, where insurance companies may distribute dividends at unrestricted payment rate. In such a scenario, the surplus level changes drastically on a dividend payday. Thus, abrupt or discontinuous changes occur due to "singular" dividend distribution policy. Moreover, the capital injections, modeled by impulse controls are exerted when surplus hits not only 0 but also a sufficiently low threshold. To maximize the performance, the impulse controls of capital injections depend on the surplus processes and can be very large, which results in a free boundary of capital injection region and adds more difficulty to analyze the optimal policies. Together with the Markov switching and the incurred claims, this gives rise to a regime-switching jump diffusion stochastic control problem with singular and impulse controls.

To find the optimal strategies, one usually solves a so-called Hamilton–Jacobi–Bellman equation. However, because of the regime-switching jump diffusion and the mixed regular and singular control formulation, the Hamilton–Jacobi–Bellman equation is in fact a coupled system of nonlinear integro-differential quasi-variational inequalities. A closed-form solution is virtually impossible to obtain. A viable alternative is to employ numerical approximations. In this work, we adopt the Markov chain approximation methodology developed in Kushner and Dupuis (2001). To the best of our knowledge, numerical methods for capital injections of regime-switching jump diffusions have not been studied in the literature to date. Budhiraja and Ross (2007) and Kushner and Martins (1991) considered singular controlled diffusions without regime switching. We focus on developing numerical methods that are applicable to find optimal controls for regime-switching jump diffusion models. A numerical algorithm for approximating optimal reinsurance and dividend payment policies under regime-switching diffusion models was developed in Jin et al. (2012). In this project, we analyze the numerical algorithm of investment strategy and dividend payment policy under a regime-switching formulation, and carried out a convergence analysis using weak convergence and relaxed control formulation of numerical schemes for singular control problems in the setting of regime switching, in which case one needs to deal with a system of quasi-variational inequalities. This paper further treats models with capital injections. As a result, we have to deal with impulse controls. Roughly speaking, due to the singular and impulse controls, the value function in each regime is verified to be a concave function and defined separately in three regions, which are capital injection region, continuation region, and dividend payment region. Taking into consideration of capital injections, the capital injections have to be ordered if the surplus violates the capital requirement for running the business. Hence, the impulse controls of capital injections will occur for sure at zero surplus. In addition, the optimization of payoff function will lead the barrier of capital injection region to be a free boundary. Thus, the impulse controls of capital injections depend on the surplus process and can be very large. These state-dependent capital injections lead to the formulation of free boundary problem, and the state-dependent "threshold" curve, as demonstrated in the numerical experiments, separates the capital injection region and continuation region. Due to the complexity of the construction, closed-form solutions are virtually impossible to be obtained and the numerical scheme thus is a viable alternative. We construct the feasible numerical approximation schemes for finding a good approximation to the underlying problems. It is worth mentioning that the Markov chain approximation method requires little regularity of the value function and/or analytic properties of the associated systems of Hamilton–Jacobi–Bellman equations, or quasi-variational inequalities, or integro-differential quasi-variational inequalities. The numerical implementation can be done using either value iterations or policy iterations.

The rest of the paper is organized as follows. A general formulation of optimal investment strategy, dividend policies, capital injections and assumptions are presented in Section 2. Certain properties of the optimal value function and the verification theorem are also presented. Section 3 deals with the numerical algorithm of Markov chain approximation method. The Poisson jumps, regular control, the singular and impulse control are well approximated by the approximating Markov chain and the dynamic programming equations are presented. Section 4 deals with the convergence of the approximation scheme. The technique of "rescaling time" is introduced and the convergence theorems are proved. Three numerical examples are provided in Section 5 to illustrate the performance of the approximation method. Finally, additional remarks are provided in Section 6.

2. Formulation

The surplus process $X(t)$ under consideration is a jump diffusion process with regime-switching under singular and impulse control. To delineate the random environment and other random factors, we use a continuous-time Markov chain $\alpha(t)$ taking values in the finite space $\mathcal{M} = \{1, \dots, n_0\}$. For each $i \in \mathcal{M}$, the premium rate is $c(i) > 0$. Let φ_n be the inter-arrival time of the nth claim, $\nu_n = \sum_{j=1}^{n} \varphi_n$. For a slightly more generality, we consider a Poisson measure in lieu of the traditionally used Poisson process. Suppose $\Gamma \subset \mathbb{R}_+$ is a compact set and the function $q(X, i, \rho)$ is the magnitude of the claim sizes, where ρ has distribution $\Pi(\cdot)$. $N(t, H) =$ number of claims on $[0, t]$ with claim size taking values in $H \in \Gamma$. Note that our formulation is general, the claim sizes are assumed to depend on the switching regime. Then the Poisson measure $N(\cdot)$ has intensity $\lambda dt \times \Pi(d\rho)$ where $\Pi(d\rho) = f(\rho)d\rho$. Assume that $q(\cdot, i, \rho)$ is continuous for each ρ and each $i \in \mathcal{M}$. At different regimes, the values of $q(\cdot, i, \rho)$ could be much different, which takes into consideration of random environment. Then the surplus process in the absence of dividend payment and investment is a regime-switching jump process given by

$$\begin{aligned} d\widetilde{X}(t) &= \sum_{i\in\mathcal{M}} I_{\{\alpha(t)=i\}}(c(i)dt - dR(t)) \\ &= c(\alpha(t))dt - \int_{\Gamma} q(X(t^-), \alpha(t), \rho)N(dt, d\rho), \end{aligned} \tag{2.1}$$

where

$$R(t) = \int_0^t \int_{\Gamma} q(X(s^-), \alpha(s), \rho)N(ds, d\rho).$$