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The 1/N investment strategy is optimal under high model ambiguity

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The 1/N investment strategy, i.e. the strategy to split one's wealth uniformly between the available investment possibilities, recently received plenty of attention in the literature. In this paper, we demonstrate that the uniform investment strategy is rational in situations where an agent is faced with a sufficiently high degree of model uncertainty in the form of ambiguous loss distributions. More specifically, we use a classical risk minimization framework to show that, for a broad class of risk measures, as the uncertainty concerning the probabilistic model increases, the optimal decisions tend to the uniform investment strategy.

To illustrate the theoretical results of the paper, we investigate the Markowitz portfolio selection model as well as Conditional Value-at-Risk minimization with ambiguous loss distributions. Subsequently, we set up a numerical study using real market data to demonstrate the convergence of optimal portfolio decisions to the uniform investment strategy.

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1. Introduction

The uniform investment strategy is interesting for researchers as well as practitioners for two reasons. Firstly, comparative studies show that naive diversification is hard to outperform as an investment strategy in a portfolio management context. Secondly, behavioral studies show that it is applied by agents in many situations. This is explained in the literature either by an inherent psychological bias, leading to potentially irrational decisions, or by the presence of some fundamental uncertainty in the decision model of the agent, making uniform diversification a rational strategy to follow. The contribution of this paper falls into the latter category as we argue that uniform diversification is an optimal strategy for certain types of risk averse investors facing model uncertainty in a stochastic programming context.

The authors do not want to imply that uniform diversification is a recommendable investment strategy in general. However, based on the results of the paper, one can explain the relative success of the 1/N rule in a stochastic portfolio optimization context as the result of an inaccurate specification of the data generating process, i.e. a lack of accuracy in the modeling of the distributions of the random asset returns. If the true model remains sufficiently ambiguous, uniform diversification may outperform more sophisticated approaches.

We start our exposition by a literature review.

The uniform investment strategy can be traced back to the 4th century, when Rabbi Issac bar Aha gave the following advice: "*One should always divide his wealth into three parts: a third in land, a third in merchandise, and a third ready to hand*".[1]

Of course, an asset allocation strategy as simple as the rule to divide the available capital evenly among some (or even all) investment opportunities falls short of the sophistication of modern portfolio theory, which in broad terms states that a portfolio should strike an *optimal* balance between the prospective return of an investment and the possible risks of investing. The optimal decision depends on the risk preferences of the investor. It can be seen as an irony that Markowitz, arguably the father of modern portfolio theory, answered the question how he manages his own funds by stating: "*My intention was to minimize my future regret. So I split my contributions fifty–fifty between bonds and equities.*" (see Zweig, 1998) – an application of the 1/N rule on an aggregate level.

In a recent paper, DeMiguel et al. (2009b) use the 1/N strategy as a benchmark in a rolling horizon setting and compare it against several portfolio optimization strategies. The models include the classical Markowitz portfolio selection rule as well as its most prominent extensions like Bayesian-Shrinkage type estimators,

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[1] Babylonian Talmud: Tractate Baba Mezi'a, folio 42a.

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aimed at dampening the effects of estimation error, and more recent approaches based on the investors beliefs about several competing asset pricing models. Furthermore, the authors include approaches that try to minimize the influence of estimation errors by restricting the asset weights or entirely focussing on the risk minimal portfolio (ignoring the expected loss dimension altogether). The results show that the benchmark $1/N$ rule outperforms most of the other more involved strategies in terms of Sharpe ratio, certainty equivalent, and turnover and is not consistently outperformed by any of the models considered in the study. The authors explain the results by stating that the errors in estimation of the parameters of the optimization models outweigh the gains of the more advanced methodology. Chan et al. (1999) and Jagannathan and Ma (2003) conduct similar studies and also conclude that it is hard to find an investment policy that consistently outperforms the uniform investment strategy. Several authors try to incorporate this finding in their proposed portfolio selection framework, see for example DeMiguel et al. (2009a) and Tu and Zhou (2011).

Apart from the success of the $1/N$ rule in empirical studies, there is evidence that uniform investment strategies are actually used in a multitude of situations where agents have to decide on a mix of different alternatives. Benartzi and Thaler (2001) conduct experiments, where subjects are asked to allocate money to different funds available in hypothetical defined contribution pension plans. The authors find that a significant share of the investors use the $1/N$ rule. This choice seems to be independent of the variety of funds offered, i.e. subjects that were offered more equity funds invested more money in equity than subjects that were confronted with an asset universe consisting of relatively fewer equity funds and more bonds. This leads the authors to the conclusion that there is a natural psychological bias towards the $1/N$ strategy, which may result in clearly irrational and even contradictory decisions. This can be interpreted as a *cognitive bias* in the sense of Tversky and Kahneman (1981) and Kahneman (2003). In Huberman and Jiang (2006), a paper motivated by the work of Benartzi and Thaler (2001), data on the choice of consumers in actual 401(k) plans is analyzed. The authors find that there is a significant share of investors (roughly two thirds) that follow the uniform investment rule. However, there is no statistical evidence of irrational behavior of the type found in the experimental studies by Benartzi and Thaler (2001).

Other studies investigating the same phenomena in different situations, under the name of *diversification heuristic*, *diversification bias*, or *variety seeking*, arrive at similar conclusions. Simonson (1990) observes variety seeking behavior in setups where multiple decisions on future consumption have to be taken as opposed to sequential decisions on immediate consumption. In Simonson and Winer (1992), an analysis of yoghurt purchases of families reveals that larger purchases (representing simultaneous decisions on future consumption) are significantly more diverse than purchases of smaller quantities by the same families. The larger purchases contain varieties which are otherwise not bought at all. The authors explain their findings by rational risk minimizing behavior of the subjects facing uncertain future preferences. On the contrary, Read and Loewenstein (1995) explain variety seeking behavior in simultaneous decisions for future consumption by cognitive deficits termed *time contraction* and *choice bracketing*. The former refers to a situation where the consumer underestimates the time between the consumption of goods and thereby overestimates the satiation effect resulting from consuming the same product, while the latter describes the phenomena that simultaneous choices are often framed as a single portfolio choice encouraging diversification.

As mentioned before, the explanations offered in the literature for the empirical prevalence of $1/N$ heuristics can be divided into papers conjecturing that there are inherent psychological patterns which encourage the use of uniform investment decisions, even in situations where it is disadvantageous, and approaches which try to find a rationale for this behavior. The latter usually refers to some kind of fundamental uncertainty about the optimization problem involved in the decision situation, making simple uniform diversification a rational strategy to follow. The contribution of this paper is to show that this is indeed the case in portfolio optimization problems under uncertainty if the distribution of the returns is ambiguous.

We consider a rational investor who tries to minimize her risk by choosing a portfolio of assets with uncertain returns. While the investor has some prior information about the possible distributions of the asset returns, the distribution is not exactly known. Hence, additional to the uncertainty about the return, there is another layer of model uncertainty present, which we will call ambiguity (also called epistemic or Knightian uncertainty after Frank Knight). Note that, this kind of uncertainty is similar to the uncertainty used as justification of the $1/N$ rule in Simonson (1990) and DeMiguel et al. (2009b) as it involves uncertainty about the nature of the optimization problem faced by the decision maker.

The investor deals with this uncertainty by adopting a worst case approach and minimizing the worst case risk under all distributions which seem plausible given the available information. In accordance with the terminology in Ben-Tal et al. (2009), we call this set of distributions the *ambiguity set*. We construct ambiguity sets as non-parametric neighborhoods of the prior in a way which is natural from a mathematical statistics' viewpoint. Subsequently, we show that under weak conditions on the risk preferences of the investor, the optimal decisions approach portfolios which obey the $1/N$ rule as the amount of model uncertainty increases.

The idea of robustifying portfolio selection problems with respect to ambiguity about the distribution of future returns is not new and is mostly pursued in the Operations Research literature. See Maenhout (2004), Calafiore (2007), Pflug and Wozabal (2007), Garlappi et al. (2007), Quaranta and Zaffaroni (2008), Vrontos et al. (2008), Kerkhof et al. (2010), Lutgens and Schotman (2010), Tarashev (2010) and Wozabal (2010) for recent advances in this direction. The proposed approaches differ in the way the ambiguity sets are defined and in the methods applied to solve the resulting optimization problems. Most of the papers make strong assumptions on the nature of the ambiguity to be able to deal with the robustified problems. Other papers that use non-parametric methods similar to our approach are Calafiore (2007), Pflug and Wozabal (2007) and Wozabal (2010). A comprehensive summary is beyond the scope of this paper.

The paper is organized as follows: in Section 2 we set up portfolio optimization problems under ambiguity and discuss how to quantify the degree of model uncertainty by the use of probability metrics. Furthermore, we discuss how the Markowitz functional as well as the Conditional Value-at-Risk fit in this framework. Section 3 contains the main theoretical results of the paper, which permit us to identify the uniform investment strategy as optimal strategy as model uncertainty increases. In Section 4, we demonstrate the theoretical results in numerical studies based on real market data. We study the ambiguous Markowitz portfolio selection model as well as the Conditional Value-at-Risk in detail. Section 5 concludes the paper by summarizing the findings as well as outlining the implications of the results.

2. Investing under ambiguity

We consider an asset universe of N financial assets with random future losses and analyze the decision problem of an agent who wants to invest a fixed amount of money in a combination of these assets for one period of time. We model the investment decision as

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