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Robust equilibrium reinsurance-investment strategy for a mean–variance insurer in a model with jumps

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HIGHLIGHTS

- A new robust mean–variance reinsurance-investment model with jumps is established.
- Different ambiguity-averse levels towards diffusion and jump risks are adopted.
- The robust equilibrium strategy and corresponding value function are derived.
- Some special cases and utility losses from model uncertainty are illustrated.
- Some interesting results and phenomena are presented.

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ABSTRACT

This paper analyzes the equilibrium strategy of a robust optimal reinsurance-investment problem under the mean–variance criterion in a model with jumps for an ambiguity-averse insurer (AAI) who worries about model uncertainty. The AAI's surplus process is assumed to follow the classical Cramér–Lundberg model, and the AAI is allowed to purchase proportional reinsurance or acquire new business and invest in a financial market to manage her risk. The financial market consists of a risk-free asset and a risky asset whose price process is described by a jump-diffusion model. By applying stochastic control theory, we establish the corresponding extended Hamilton–Jacobi–Bellman (HJB) system of equations. Furthermore, we derive both the robust equilibrium reinsurance-investment strategy and the corresponding equilibrium value function by solving the extended HJB system of equations. In addition, some special cases of our model are provided, which show that our model and results extend some existing ones in the literature. Finally, the economic implications of our findings are illustrated, and utility losses from ignoring model uncertainty, jump risks and prohibiting reinsurance are analyzed using numerical examples.

1. Introduction

The application of stochastic control theory to the optimal reinsurance-investment problem has been the focus of a good part of actuarial research, and the interest in this problem continues to grow. In recent years, this problem has been studied in terms of a variety of objectives, such as minimizing the probability of ruin (see Promislow and Young, 2005; Azcue and Muler, 2013), maximizing the expected utility from terminal wealth (see Bai and Guo, 2010; Liang and Yuen, 2016), and the mean–variance criterion (see Pressacco et al., 2011; Bi et al., 2013).

Although the optimal reinsurance-investment problem has been widely investigated by many scholars, only a few have incorporated the model uncertainty into it. However, it is a notorious fact that the return of risky assets is difficult to be estimated with precision. Thus, some scholars have advocated and investigated the effect of model uncertainty on portfolio selection. Robust decision making in the portfolio context is introduced by Maenhout (2004). Maenhout (2004, 2006) investigates the effect of ambiguity on the intertemporal portfolio choice in a setting with constant investment opportunities and in a setting with a mean-reverting equity risk premium, respectively. A number of other papers are built on Maenhout (2004) to

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address the implications of ambiguity on portfolio choice. Liu (2010) examines the robust consumption and portfolio choice for time-varying investment opportunities. Flor and Larsen (2014) determine the optimal investment strategy for an ambiguity-averse investor with a stochastic interest rate. Munk and Rubtsov (2014) introduce a stochastic interest rate and inflation into a portfolio management problem for an ambiguity-averse investor. Yi et al. (2015b) focus on an optimal portfolio selection problem with model uncertainty in a financial market that contains a pair of stocks. From these papers, we find that compared with making ad-hoc decisions about how many errors are contained in the estimates for the parameters of risky assets, investors may consider alternative models that are close to the estimated model. This method has also been accepted in the robust optimal reinsurance-investment problem. Lin et al. (2012) and Korn et al. (2012) investigate the optimal reinsurance problem or the optimal reinsurance-investment problem with model uncertainty by using a stochastic differential game approach. Yi et al. (2013) and Yi et al. (2015a) study the problem of robust optimal reinsurance-investment for an ambiguity-averse insurer (AAI) under the expected exponential utility maximization and mean–variance criteria, respectively. Pun and Wong (2015) consider the problem of robust optimal reinsurance-investment with multi-scale stochastic volatility using a general concave utility function.

However, most of the literature on the robust optimal reinsurance and investment problem assumes that the AAI's surplus process and the risky asset's price process follow the diffusion model, which ignores the significant effect that jumps have on the optimal strategy. As is mentioned in Branger and Larsen (2013) and Aït-Sahalia and Matthys (2014), there are pronounced differences between ambiguity aversion with respect to (w.r.t.) diffusion and jump risks. Therefore, in the portfolio selection problem, ignoring ambiguity w.r.t. the jump risk may result in large losses in the financial market. In this paper, we consider the optimal reinsurance-investment problem for an AAI who faces uncertainties regarding models in the financial and insurance markets with jumps.

To the best of our knowledge, no published work addresses the robust optimal reinsurance-investment problem with jumps under the mean–variance criterion for an AAI. Traditional dynamic mean–variance optimization problem is a time-inconsistent problem, and most of the literature derives an optimal strategy that is only optimal at the initial time. However, time consistency of strategies is a basic requirement for rational decision making in many situations. A decision maker sitting at time t would consider that, starting from $t+\Delta t$, she will follow the strategy that is optimal sitting at time $t+\Delta t$. Namely, the optimal strategy derived at time t should agree with the optimal strategy derived at time $t+\Delta t$. Because the time-consistency of strategies is important for a rational decision-maker, recently many scholars have developed a time-consistent strategy for the dynamic mean–variance asset allocation problem. The main approach is to formulate the problem within a non-cooperate game theoretic framework, with one player for each time t, where player t can be regarded as the future incarnation of the insurer at time t. Then we aim to derive the equilibrium strategy of the game. For more details, we refer the reader to Björk and Murgoci (2010), Zeng et al. (2013), Li and Li (2013), Björk et al. (2014) and references therein. In our model, the insurer's surplus process is assumed to follow the classical Cramér–Lundberg (C–L) model, and the insurer is allowed to purchase proportional reinsurance or acquire new business and invest in a financial market to manage her risk. The financial market consists of a risk-free asset and a risky asset whose price process is described by a jump-diffusion model. Given that the market (true model) may deviate from the estimated model (reference model) in reality, we incorporate model uncertainty into our model and assume that the insurer is ambiguity-averse about diffusion and jump risks. On the basis of the above setup and by applying stochastic control theory, we formulate a robust optimization problem with alternative models and establish the corresponding extended Hamilton–Jacobi–Bellman (HJB) system of equations. Furthermore, we derive both the robust equilibrium reinsurance-investment strategy and the corresponding equilibrium value function. Some special cases of our model are also provided, which show that our model and results extend some ones in the existing literature. Finally, the economic implications of our findings are illustrated, and utility losses from ignoring model uncertainty, jump risks and prohibiting reinsurance are analyzed using numerical examples. The main contributions of this paper are as follows: (i) a new optimal reinsurance-investment model incorporating model uncertainty and jumps under the mean–variance criterion is established; (ii) the robust equilibrium strategy and the corresponding equilibrium value function are derived explicitly, and our model and results extend some ones in the existing literature; and (iii) utility losses from ignoring model uncertainty, jump risks and prohibiting reinsurance for the AAI are analyzed, and some new findings are provided.

The remainder of this paper is organized as follows. Section 2 describes the formulation of the model. Section 3 derives the explicit expressions of the robust equilibrium reinsurance-investment strategy and the corresponding equilibrium value function, and provides some special cases of our model. Section 4 presents some numerical examples to illustrate our results and sensitivity analysis of utility losses. Section 5 concludes this paper.

2. General formulation

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$ be a filtered complete probability space satisfying the usual condition, where $T > 0$ is a finite constant representing the investment time horizon; $\mathcal{F}_t$ stands for the information available until time t; and $\mathbb{P}$ is a reference measure. Any decision made at time t is based on $\mathcal{F}_t$, and all stochastic processes below are supposed to be well-defined and adapted to this probability space. In addition, suppose that there are no transaction costs or taxes in the financial and insurance markets, and trading can be continuous.

Suppose that an insurer's surplus process follows the classical C–L model. In this model, without reinsurance and investment, her surplus process is described by

$$R(t) = x_0 + ct - \sum_{i=1}^{N_1(t)} Z_i,$$

where $x_0 \geq 0$ is the initial surplus; c is the premium rate; and $\sum_{i=1}^{N_1(t)} Z_i$ is a compound Poisson process, representing the cumulative claims up to time t, $\{N_1(t)\}_{t\in[0,T]}$ is a homogeneous Poisson process with intensity $\lambda_1 > 0$, and the claim sizes $Z_1, Z_2, \ldots$, independent of $N_1(t)$, are assumed to be independent and identically distributed (i.i.d.) positive random variables with common distribution $F(z)$, finite first moment $\mathrm{E}[Z_i] = \mu_Z$ and second moment $\mathrm{E}[Z_i^2] = \sigma_Z^2$. Furthermore, we assume that the premium rate c is assumed to be calculated according to the expected value principle, i.e., $c = (1+\theta)\lambda_1\mu_Z$, where $\theta > 0$ is the safety loading of the insurer.

In addition, we assume that the insurer can control her insurance risk by purchasing proportional reinsurance or acquiring new business, such as acting as a reinsurer of other insurers (see Bäuerle, 2005). For each $t \in [0, T]$, the proportional reinsurance/new business level is denoted by the value of risk exposure $p(t) \in [0, +\infty)$. When $p(t) \in [0, 1]$, it corresponds to a proportional reinsurance cover. In this case, the insurer diverts parts of the premium to the reinsurer at the rate of $(1+\eta)(1-p(t))\lambda_1\mu_Z$,