Decision Support Systems 54 (2013) 816–825

Contents lists available at SciVerse ScienceDirect

Decision Support Systems

journal homepage: www.elsevier.com/locate/dss

Algorithmic determination of the maximum possible earnings for investment strategies

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ARTICLE INFO

Article history:
Received 4 June 2012
Received in revised form 9 August 2012
Accepted 9 September 2012
Available online 5 October 2012

Keywords:
Optimization
Algorithm
Graph
Investment strategy

ABSTRACT

This paper proposes a new method for determining the upper bound of any investment strategy's maximum profit, applied in a given time window [0,T]. This upper bound is defined once all the prices are known at time T and therefore represents the ex-post maximum efficiency of any investment strategy determined during the relevant time interval. This approach allows us to gauge in absolute terms those behaviors defined through atomic "buy" and "sell" actions, and can be extended to more complex strategies. We show that, even in the ex-post framework, establishing this upper bound when transaction costs are implemented is extremely complex. We first describe this problem using a linear programming framework. Thereafter, we propose to embed this question in a graph theory framework and to show that determining the best investment behavior is equivalent to identifying an optimal path in an oriented, weighted, bipartite network or a weighted, directed, acyclic graph. We illustrate this method using real world data and introduce a new theory about absolute optimal behavior in the financial world.

1. Introduction

Performance gauging in Finance is a complicated issue that generates a series of methodological questions (Jensen [10], Sharpe [19], Elton et al. [6] or Malkiel [14]). In assessing the performance of a sequence of investment/divestment actions relating to a financial asset over time (for example a particular tracker fund), two frameworks can be considered.

The first option is to adopt an ex-ante evaluation point of view, answering the following question: "*Were the choices of the investor, given his knowledge of the future at that time, optimal or not when they were realized*?". This point of view acknowledges that investment occurs in a stochastic context and that a poor ex-post result does not necessarily indicate that bad decisions were made ex-ante, or during the decision process. Notice that this ex-ante performance assessment requires an awareness of the investor's conception of the future at each stage in the process, and is therefore difficult to achieve in practice.

The second option is to adopt an ex-post evaluation approach, which considers only the statistical result of a given investment strategy over time, once price motions are perfectly known. This approach is widely used in professional asset management. For example, the performance of various *investment styles* is gauged using this technique. Financial journals use this ex-post approach to create yearly rankings and to report on the performance of asset managers and funds. In the latter case, performance is evaluated using a relative comparison among funds, as it is impossible to know what would have been the best behavior during the relevant period, or how the best output compares with the performance upper bound.

This paper can provide, in the ex-post framework previously described, the upper bound to any investment strategy in a given time window, for the trading of a single financial asset. We do not address strategic/tactical allocation questions (for the use of decision support systems in this context, see Beraldi et al. [1]) or the operational process that allows fund managers to identify states in the market where buying or selling is particularly appropriate (for example in exploiting results delivered by neural network forecasting, see Lam [11] or Chen and Leung [2], rough sets Shen and Loh [20] or stock cherting Leigh et al. [12]). Neither do we propose a method that ranks various strategies in terms of risk-return performance (although our approach might be extended to this bicriteria framework; for an example of heuristic algorithm allowing to optimize complex risk-return investment problems, see for instance Liu et al. [13]). Instead, we offer a computational characterization of the profits upper bound that might have been reached, by chance or skill, in trading a single financial asset during a given time-window.

Computing this limit allows the determination of an ex-post optimal strategy S_* that actually delivers the upper bound. We call this problem the S_*-determination, and show that it is far from trivial,

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0167-9236/$ – see front matter
http://dx.doi.org/10.1016/j.dss.2012.09.020

despite its similarity to many popular models that have frequently proved completely inefficient. Our new method delivers *an absolute performance indicator* geared towards the ex-post evaluation of a wide range of trading strategies.

This upper bound can be characterized using a linear programming framework and solved with a simplex approach or with dynamic programming formalism. Nevertheless, if these methods are theoretically correct, they suffer from severe limitations in terms of computability (in the worst case, the underlying algorithm being non-polynomial for the simplex). We therefore propose to embed this question in a graph theory framework and to show that determining the best investment behavior is equivalent to identifying an optimal path in an oriented, weighted, bipartite network. We illustrate these results with real data as well as simulated algorithmic trading methods.

This paper is organized as follow. We first formalize the framework we start from, define explicitly the S_*-determination problem, and give some illustrations of the complexity of the optimization task delivering S_* (Section 2). We then present the mathematical frameworks related to these questions (Section 3) and a new algorithm geared at identifying the S_* strategy (Section 4). In the last section we illustrate this latter algorithm and provide some practical implementations to gauge the ex-post absolute performance of a few trading strategies (Section 5).

2. Elements of the game, formalizations and examples

2.1. Elements of the game

Consider the situation in which one investor has realized a sequence of investments/divestments for a given financial asset (a stock, an index or a portfolio) during a given time window $[t=0, t=n]$. At time $t=n$, her actions (for example Buy, Sell) and the prices at which they were undertaken (that is, the historical price series $\vec{p}=\{p_t|t\in[0,n]\}$) are perfectly known. We do not focus on "how" the investor behavior has been formed (for example, this investor should have generated trading rules with genetic programming, see Potvin et al. [17]), or on the relevant information that are needed to do so. We rather focus on the decisions it delivered as *data* and that lead to a specific profit (or loss) at time $t=n$.

This investor has the opportunity to assess her performance with respect to the best possible behavior in this time window. This assessment can be made checking whether or not her behavior matches the absolutely optimal set of actions that could have had realized. Notice this optimal set can theoretically be computed at time $t=n$ since all the prices are known.

This comparison requires some hypotheses to be respected. The following "rules of the game" present these hypotheses and describe a formal framework in which the actual set of undertaken, compared actions can be matched against any other set of trades pertaining to the same conditions, and specifically, to the absolutely optimal set of actions.

2.1.1. Market liquidity

Let's assume that the prices in $[t=0,t=n], n\in\mathbb{N}$ are those at which this investor has had the opportunity to rebalance her portfolio. We posit a price-taker framework, *i.e.*. The agent's decisions cannot affect these prices; sufficient liquidity at these prices is assumed.

2.1.2. The "all or nothing" general constraint

We now define a set of "rules" for this investor, in other words, a series of constraints on her behavior. These simplifications are useful in allowing rigorous comparisons between sets of actions (strategies) undertaken during a given period. In this article, these rules define an "all or nothing" behavior: whether the investor is totally invested in the risky asset or has realized all her wealth in cash:

- At the initialization stage (*i.e.* at $t=0$), the initial wealth W_0 of the investor is composed of a certain amount of cash (C_0) and no stock ($A_0=0$): $W_0=A_0+C_0$. At date $t=1$ (the beginning of the game) we posit C_1 to be equal to the first price of the considered time series.
- The investor must decide for each $t\in(1,n)$ one specific action with regard to the composition of her portfolio: Buy, Sell or Remain unchanged (respectively coded B, S and U). In other terms, the investor has to compose a "sentence" of size n using characters in B, S,U. The interpretation of each of these actions is as follows:
  - Buy: One can write B *if and only if* $W_{t-1}=C_{t-1}$. If B is written at date t; all the investor's cash is converted into assets (delivering a new quantity for $A_t\neq 0$), assuming transaction costs at a c% rate,

    $$A_t = \frac{W_{t-1}}{p_t \times (1+c)}$$

    Additionally, the first character in any sentence must be a B.
  - Sell: *if and only if* $A_{t-1}\neq 0$, the investor can write S and convert her position into cash. Considering an identical rate of transaction costs c,

    $$C_t = A_{t-1} \times (p_t \times (1-c))$$

  - Remain unchanged: Whatever the nature of W_{t-1} (cash or assets), she can also decide to write U and let her position remain unchanged at date t: $W_t=W_{t-1}$.
- This "sentence" is one investment strategy S_i over $\vec{p}$ chosen in a set of strategies $\{S\}$. Notice, that in this framework $Card\{S\}=2^n$.

Note that these "rules of the game" can be used by the investor without knowing the future prices (she performs ex-ante decisions by definition) and will deliver different results: each instance of S_i can be gauged in terms of relative performance with respect to any other strategy $S_{j,j\neq i}$ (and reciprocally). Among these strategies, the best possible one in terms of maximum profit, denoted by S_*, can be determined ex-post the realization of the price sequence (when $t=n$). Consequently, the objective function is:

$$S_* \rightarrow max(W_{t+n}-W_t) \tag{2.1}$$

Thus, it can be generated by an investor acting in the "ex-ante" framework by chance or skill (the latter alternative is not discussed here). In any case, S_* is the upper bound in terms of absolute performance in $\{S\}$ and therefore a much more interesting parameter for gauging any strategy S_i. As we will show later, the best strategy is relatively easy to identify when transaction costs are not implemented. When transaction costs alter profits, this identification is far more complex.

2.2. Basic illustration

Let's consider the following (arbitrarily chosen) price series (see Table 1 and Fig. 1):

This example illustrates simply that, when transaction costs are minor (or absent), the best strategy consists in accumulating all

Table 1
Basic artificial time series.

t	1	2	3	4	5	6	7	8	9	10	11	12
p_t	100	120	90	160	126	150	140	160	110	170	168	180