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Optimal investment strategies in an international economy with stochastic interest rates ☆

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ABSTRACT

We investigate how investors should optimally choose to invest in a dynamically complete international market. We find closed-form solutions for the optimal investment strategy and for the wealth loss an investor suffers from not investing internationally. Theoretically, we show that the gain from international investment is due to the speculative investment only, and why it is important for an investor from a large economy to invest in a small economy. In a numerical example we compare the wealth losses investors from Denmark and the U.S. suffer due to home bias.

1. Introduction

It is a well-known fact that an investor can benefit from investing internationally instead of investing only domestically, see among others Santis and Gerard (1997). In spite of this fact, the papers by Cooper and Kaplanis (1994) and French and Poterba (1991) document that investors hold a disproportionately high share of their wealth in domestic assets. How much does the investor actually lose from not investing internationally? This is one of the questions we try to answer in this paper. Furthermore, since the papers by Merton (1971, 1973) it has been recognized that long-term investors should hedge stochastic changes in investment opportunities, such as changes in interest rates, excess returns, volatilities, and inflation rates. Another question is therefore: How should investors optimally choose their investment strategy in an international economy with a stochastic opportunity set? In this paper we come up with a model where an individual investor has access to the domestic as well as a foreign market, and where the interest rates in both markets are stochastic. We then investigate how investors with constant relative risk aversion preferences for terminal wealth should optimally choose their investment strategy.

We consider an international setup where trading takes places continuously in time. The state of the economy is fully described by the term structures of interest rates in the two countries. The term structures are assumed to follow a two-factor dynamic affine Gaussian model. The model has the important property that the interest rates will be correlated and, furthermore, the drifts in the domestic and the foreign short rates will be interdependent. This way of modeling the term structures is similar to the equilibrium model developed by Nielsen and Saá-Requejo (1993), but with the essential exception that we assume a Gaussian setup, while

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Nielsen and Saá-Requejo (1993) consider a setup with square root processes. The link which facilitates trade between the two economies is the real spot exchange rate, which is assumed to be log-normally distributed. In line with other studies on dynamic portfolio choice, our framework is a partial equilibrium setup with exogenous given processes for the interest rates, currency, and stock prices. Hence, we ignore international equilibrium considerations.

The affine structure in our model allows us to find a closed-form solution for the investor's optimal investment strategy, and hence gives us the possibility of making a quantitative analysis of the optimal investment strategy in an international setup. This is, to our knowledge, the first paper which considers a concrete model in an international asset allocation setup. The paper by Lioui and Poncet (2003) considers optimal international portfolio allocation in a general multi-period model. The paper posits an international economy where real exchange rates, real interest rates and real stock price changes follow general stochastic processes whose drifts and diffusion parameters are driven by an arbitrary number of state variables. But due to the generality of the paper they do not come up with a quantitative analysis of the optimal investment strategy, as we do in this paper. We find, as in Merton (1973), that the optimal investment strategy can be decomposed into two portfolios, a speculative portfolio and a hedge portfolio, where the latter portfolio describes how the investor should optimally hedge against the stochastic changes in the investment opportunities. We show that the hedge portfolio is the same for an investor who has access to the international market and an investor who has access to the domestic market only, i.e. the exchange rate risk is not hedged per se.[1] Hence, the gain from international investment is due to the speculative investment only.

Furthermore, the affine structure in our model allows us to find closed-form solutions for the wealth loss an investor suffers from only investing domestically. Due to the larger investment opportunity set the investor gets a higher expected utility if he makes in both domestic and foreign investments, instead of investing only domestically. This gain from international investment can be determined as the compensation in the shape of extra initial capital a purely domestic investor must have to obtain the same expected utility as with the original initial capital invested internationally. The closed-form solution for the loss makes it possible for us to perform a quantitative analysis of the loss. If the loss is relatively small, the investor will have no particular advantage from investing internationally, which would explain the above-mentioned empirical observation that many investors only invest a small part or nothing of their wealth in foreign assets and instead invest in domestic assets. To our knowledge, this is the first paper which quantifies the importance of investing internationally.

Two cases of the domestic versus the international market will be considered. In the first case we assume that a small change in the short rate of the foreign country does have an effect on the short rate of the domestic country. For instance, this would be the case if we considered an example with the United States and the United Kingdom. In the second case we assume that a small change in the short rate of the foreign country does not have any effect on the short rate of the domestic country. This would be the case if the investor's home country is a big economy like the United States and the foreign country is a small economy like Denmark. The two cases then differ in the available investment assets. An investor in case 1 gains from investing internationally due to the foreign stock market only, while an investor in case 2 gains from investing internationally both due to the foreign bond and stock markets. Hence, an investor of a large economy should invest in bonds of small economies to pick up the corresponding risk premium, whereas this is not the case for an investor of a small economy. However, as we see in the numerical example, this does not necessarily mean that a case 1 investor will suffer a larger loss than a case 2 investor. Which investor suffers the largest loss is due to the size of the associated risk premiums for the assets they can trade in.

We provide an application of the model by calibrating it to historical data and examining its implications for optimal portfolio weights and the wealth loss an investor suffers due to home bias. We consider the United States and Denmark as the two countries in our economy. We assume that a small change in the Danish short rate does not have any effect on the U.S. short rate, while a small change in the U.S. short rate does have an impact on the Danish short rate. This implies that the U.S. investor gains from investing internationally both due to the Danish bond and stock market, while the Danish investor only gains from investing internationally due to the U.S. stock market. We find that the loss a U.S. investor with a risk aversion of 5 or higher suffers from not investing internationally seems insignificant. For instance, an investor with an investment horizon of 30 years and a risk aversion of 6 will suffer a loss of approximately 8% of initial wealth. For a risk aversion of 10 this loss reduces to 5%. For more risk tolerant investors we find that the investor will suffer a significant loss. On the other hand, we find that a Danish investor suffers a much larger loss. For instance, a Danish investor with a risk aversion of 6 and a 30-year investment horizon suffers a loss of approximately 43% due to home bias. Hence, we find that it is much more important for the Danish investor to invest internationally than it is for a U.S. investor.

Since the original work of Merton (1971) there have been a number of papers on optimal investment strategies in the presence of richer stochastic environments. Some of these are Munk, Sørensen and Vinther (2004), and Brennan, Schwartz and Lagnado (1997), who consider asset allocation with stochastic interest rates and predictability in stock returns, Sørensen (1999), Brennan and Xia (2000), Wachter (2003), Munk and Sørensen (2004), and Liu (2007) look at a setup with stochastic interest rates, while Liu, Longstaff and Pan (2003) look at a setup with event risk. The main difference between these and our paper is that we allow the investor to trade in an international market. The papers by Ang and Bekaert (2002) and Das and Uppal (2004) consider a setup with time-varying correlations in an international economy. Das and Uppal (2004) look at an economy where the international equities are characterized by jumps, and develop a theoretical framework along the lines of Merton (1971) in contrast to Ang and

[1] This is in line with the findings in the paper by Nielsen and Vassalou (2006), who show in a very general setup that investors with time-additive utility functions will only hedge stochastic variations in the short-term interest rate and in the squared market prices of risk.