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Demographic structure and capital accumulation: A quantitative assessment

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A R T I C L E I N F O

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A B S T R A C T

In a recent paper, d'Albis [2007. Demographic structure and capital accumulation. Journal of Economic Theory 132, 411–434] shows that the effect of population growth on capital accumulation is ambiguous in overlapping-generations models with age-specific mortality rates, contrasting to the predicted negative effect in Diamond [1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150] and Blanchard [1985. Debt, deficits, and finite horizons. Journal of Political Economy 93, 223–247]. The quantitative exercises of this paper indicate that while in principle a positive relation between population growth and capital accumulation is possible, this relation is practically always negative for industrial countries. Intuition based on capital dilution and aggregate saving effects is provided. This paper also complements d'Albis [2007. Demographic structure and capital accumulation. Journal of Economic Theory 132, 411–434] in characterizing the steady-state equilibrium in more familiar economic concepts.

1. Introduction

Motivated by the lack of robust evidence of demographic impact on asset prices such as the interest rate (Poterba, 2001), d'Albis (2007) develops an analytically tractable overlapping-generations model with life-cycle saving motive and a general mortality pattern. He shows that the effect of a change in the population growth rate (through fertility change) on capital accumulation may be positive or negative.[1] The result contrasts sharply to the predicted negative relation between population growth and capital accumulation in two most well-known overlapping-generations models – the two-period overlapping-generations model (Diamond, 1965) and the continuous-time overlapping-generations model with age-invariant mortality (Blanchard, 1985).

The approach in d'Albis (2007) is mainly theoretical, and he uses a model with age-specific mortality rates, a general production function with no technological progress, and age-varying discount rates. Moreover, d'Albis (2007) follows Blanchard (1985) in assuming that the agents do not retire. To deliver sharper predictions, this paper complements his analysis by providing quantitative assessment regarding the relation between fertility change and capital accumulation in industrial countries. As such, this paper uses empirical age-specific mortality rates of USA and a constant discount rate, allows for technological progress, and chooses as values of production and preference parameters those commonly used in other studies. Two versions of the overlapping-generations model are considered in this paper. First, this paper examines a

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[1] d'Albis (2007) does not consider mortality change and shows that in such an environment, a rise in the birth rate always leads to an increase in the population growth rate. This paper also does not consider mortality change, and uses the terms 'population change' and 'fertility change' interchangeably.

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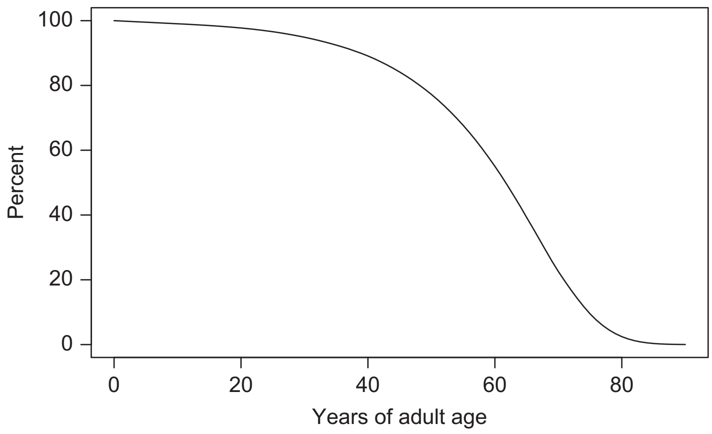

Fig. 1. Percent surviving (United States, 2004).

model without retirement, so as to facilitate comparison with d'Albis (2007). It then incorporates the empirically relevant feature of retirement and considers the case in which the workers retire.

Two major conclusions arise from the quantitative exercises. First, when the population growth rate increases from, say, −6% to 2% in the model with no retirement, its effect on the steady-state capital is first positive and then negative. This result gives some support to the general prediction in d'Albis (2007). Second, when the population growth rate changes within the more relevant interval from about −2% to 2%, the effect on capital accumulation is monotonically negative in both versions of the model (with or without retirement). Overall, the results of this paper suggest that while a positive relation between population growth and capital accumulation is possible, their relation is practically always negative for the parameters relevant for industrial counties. In this sense, the prediction of the highly stylized overlapping-generations models of Diamond (1965) and Blanchard (1985) regarding the sign of this relation is not misleading.

The paper is organized as follows. Section 2 discusses the model. Section 3 characterizes the steady-state equilibrium of the model, and discusses the existence and uniqueness of the equilibrium. Section 4 examines the effects of fertility changes on steady-state capital accumulation in an environment without retirement. Section 5 examines the same issues in an environment with retirement. Section 6 discusses how the model can be extended to examine the impact of social security changes. Section 7 provides concluding remarks.

2. The model

This paper considers a continuous-time overlapping-generations model, modified from Tobin (1967), Blanchard (1985), Bommier and Lee (2003), and d'Albis (2007). This section gives a brief description of the model; a more detailed discussion can be found in the above papers, especially d'Albis (2007). The demographic and labor supply features of the model will be discussed first, to be followed by a discussion of the consumption decision of individual cohorts.

Represent the probability that an individual survives to at least age x by the survival function $l(x)$, where $x \in [0, \Omega]$, Ω is the maximum age, and $l(0)$ is normalized to 1. Since most overlapping-generations models assume that the adult stage starts at age 20, in the following analysis x is interpreted as adult age, which is defined as actual age minus 20. Using the life table information for the USA (men and women combined) in 2004, obtained from the Human Mortality Database (2007), one can calculate the survival probability in terms of adult age according to $l(x) = l_{\text{actual}}(x+20)/l_{\text{actual}}(20)$, where $l_{\text{actual}}(.)$ is the survival probability based on actual age.[2] The survival function $l(x)$ in USA 2004 is shown in Fig. 1.

The instantaneous mortality rate at age x, $\mu(x)$, is related to $l(x)$ by

$$\mu(x) = -\frac{1}{l(x)}\frac{\mathrm{d}l(x)}{\mathrm{d}x}. \tag{1}$$

In the following analysis, the age-specific mortality rates are assumed to be time-invariant and Ω is taken to be 90 (since maximum age is 110 in the US life table).

It is also assumed, implicitly, that age-specific fertility rates are time-invariant. However, since fertility decisions are usually not modelled in the framework used in Blanchard (1985) and d'Albis (2007), this paper simply uses the alternative,

[2] It is more convenient to express age as a continuous variable in the following analysis, even though the life table data are available only for discrete years of age (and thus, the quantitative exercises are based on discrete years of age). Moreover, while it is more appropriate to use cohort survival information, this paper uses a period life table because cohort life tables of people born recently are not available.