

# Equilibrium dynamics in an aggregative model of capital accumulation with heterogeneous agents and elastic labor

Cuong Le Van\*, Manh-Hung Nguyen, Yiannis Vailakis

*Centre d' Economie de la Sorbonne, Université Paris-1, Pantheon-Sorbonne,  
CNRS, 106-112 Bd de l' Hôpital, 75647 Paris Cedex 13, France*

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## Abstract

The paper extends the canonical representative agent Ramsey model to include heterogeneous agents and elastic labor supply. The welfare maximization problem is analyzed and shown to be equivalent to a non-stationary reduced form model. An iterative procedure is exploited to prove the supermodularity of the indirect utility function. Supermodularity is subsequently used to establish the convergence of optimal paths. © 2006 Elsevier B.V. All rights reserved.

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## 1. Introduction

Optimal growth theory is useful in qualitatively characterizing simple dynamical systems and in providing constructive methods for the quantitative analysis of the solutions to more complex ones. The usefulness is, for some purposes, enhanced because of the intimate connections between optimal growth theories and their equilibrium counterparts. In a decentralized economy, we seek knowledge about the time paths of the various prices for goods and production factors as well as the distribution of income and wealth. Dynamic optimization techniques

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\* Corresponding author.

*E-mail addresses:* [levan@univ-paris1.fr](mailto:levan@univ-paris1.fr) (C. Le Van), [manh-hung.nguyen@malix.univ-paris1.fr](mailto:manh-hung.nguyen@malix.univ-paris1.fr) (M.-H. Nguyen), [vailakis@univ-paris1.fr](mailto:vailakis@univ-paris1.fr) (Y. Vailakis).

used extensively in growth theory facilitate the study of the evolution of those economic aggregates.

A major concern in the area of optimal growth has been the analysis of the short-run and asymptotic behavior of optimal solutions. At issue are questions concerning the existence and asymptotic stability of optimal programs with respect to the stationary optimal stock (turnpike results) as well as the possibility of cyclical or even chaotic behavior.

One-sector representative agent models where utility is derived solely from consumption have been studied extensively in the literature under a variety of different technological specifications. A well known property of those models is the monotonicity of the optimal capital path. This property is persistent even when the technology exhibits increasing returns and investment is irreversible (see Dechert and Nishimura, 1985; Le Van, Morhaim and Dimaria, 2002; Duran and Le Van, 2003). Thus, it is often suggested that one-sector models display simple dynamics.

Becker and Foias (1987) show that agents' heterogeneity plays a crucial role to the appearance of nonmonotonic dynamics in a single-sector model. Studying a specific economy with incomplete markets as represented by borrowing constraints, they demonstrate that deterministic cycles of period 2 may occur. Becker and Foias (1994), discuss in more detail the issue of equilibrium cycles and their construction using bifurcation analysis. Their work has been further elaborated by Sorger (1994).

In a different setting (i.e. complete market structure), Le Van and Vailakis (2003) have also shown that the monotonicity property does not carry over if one permits many consumers with different discount factors. The model does not exhibit cyclical behavior. The convergence of the optimal capital sequence to a particular stock  $k^s$  is still true, but that stock is not itself a steady state.<sup>1</sup> This result implies that the optimal capital sequence initiated at  $k_0 = k^s$  converges to  $k^s$  in the long-run, but it is not a constant sequence. Hence, the resulting optimal capital path cannot be monotonic. The model exhibits the *twisted turnpike property* (see Mitra, 1979; Becker, 2005): the optimal capital accumulation paths starting from different initial capital stocks converge to each other or come together in the limit, but this limit is not itself an optimal stationary program. This is a fundamental property of the heterogeneous agent model and it shows one way in which this model differs significantly from its representative agent counterpart.

The purpose of this paper is to examine whether and under which conditions similar properties hold when the model studied in Le Van and Vailakis (2003) is extended to include an endogenous non-reproducible factor such as labor.

The analysis in Le Van and Vailakis (2003) is carried out by exploiting the so called reduced form model associated with the welfare maximization problem. The presence of heterogeneous discount factors turns out the reduced-form problem to be nonstationary, making the issue of convergence of optimal paths a nontrivial one. Their argument exploits the fact that the indirect utility function  $V_t$  associated with the reduced form model is  $C^2$  in the interior of a set describing the feasible activities in period  $t$ . This allows them to use the differentiable characterization of supermodularity to show that  $V_t$  is supermodular. The supermodularity of  $V_t$  then implies that the stationary problem involving the agents with a discount factor equal to the maximum one, has a unique stable steady state  $k^s$ . Exploiting additional properties of optimal paths, they subsequently show that the optimal capital sequence associated with the initial problem involving all agents converges to  $k^s$ .

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<sup>1</sup> An early turnpike result for multisector economies with heterogeneous agents has been proved by Coles (1985).

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