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Optimal capital accumulation under price uncertainty and costly reversibility

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ABSTRACT

We consider the optimal capital accumulation policy of a competitive firm operating in the presence of decreasing returns to scale, price uncertainty, and costly reversibility of investment. We characterize the optimal accumulation policy and derive the value of the firm by focusing on the marginal investment decision and solving the associated optimal timing problem characterizing the option value of the associated opportunity to either disinvest or acquire a marginal unit of capacity. We also characterize the required exercise premia associated with the optimal policies and demonstrate that hysteresis prevails within this class of accumulation problems as well.

1. Introduction

One of the basic lessons of the literature, considering the optimal capital accumulation of a competitive price-taking firm, is that irreversibility results into an accumulation rule which differs from the standard myopic investment rule requiring that the productive stock should be maintained at a level where its marginal revenue product coincides with its marginal user cost (cf. Arrow, 1968; Nickell, 1974a, 1974b; Baldwin, 1982; Bernanke, 1983; Pindyck, 1988, 1991; Bertola and Caballero, 1994; Dixit, 1995; Caballero and Leahy, 1996; Bertola, 1998; for comprehensive textbook treatments of this subject, see Nickell, 1978; Dixit and Pindyck, 1994). The main reason for this observation is that, in the presence of irreversibility, the firm is unable to instantaneously adjust the productive capacity to a desired optimal level should market conditions unexpectedly deteriorate and change in an unfavorable direction after an investment decision has been made. As intuitively is clear, the presence of uncertainty pronounces this effect and raises the required investment premium associated with the irreversible decision by increasing the option value of waiting.

Even though the literature on irreversible capital accumulation in the presence of uncertainty is extensive, the impact of partial reversibility of investment on the optimal capital accumulation policies has not been investigated to the same extent. In this respect, the papers by Abel and Eberly (1996) and Abel et al. (1996) constitute the pioneering studies on this subject. Abel and Eberly (1996) considered optimal investment in the presence of uncertainty and costly reversibility by modeling the underlying driving stochastic factor dynamics as geometric Brownian motion and assuming that the operating profit flow of the

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firm is homogenous of degree one as a function of the underlying price and the installed capital stock. They proved that the optimal investment policy is characterizable as an accumulation rule requiring that the marginal revenue product of capital should be maintained between two optimal thresholds at all times. Abel et al. (1996), in turn, considered a general discrete two-period investment model, where both expanding the current productive capital stock and disinvestment are costly. They demonstrated how the optimal investment policy can be characterized in terms of embedded put and call options, and showed that the relative sizes of these options determine the net effect of expandability and reversibility on investment.

It is worth emphasizing that costly reversibility also arises in studies considering optimal investment in a competitive equilibrium (cf. Leahy, 1993; Baldursson and Karatzas, 1997; for an analysis of the problem in an oligopolistic setting, see Baldursson, 1998). Within that modeling framework, the industry equilibrium can be characterized as the solution of a singular stochastic capital accumulation problem subject to costly reversibility (by the assumed absence of arbitrage condition guaranteeing that increasing the stock is more expensive than selling out capital). The principal finding of these studies is that myopic behavior is optimal in a competitive equilibrium, and, therefore that the optimal accumulation rule in the equilibrium can be derived by modeling the driving price process as exogenous and independent of industry output.

In this paper, we reconsider the problem originally addressed by Abel and Eberly (1996) and Abel et al. (1996), and analyze how price uncertainty and costly reversibility affects the optimal accumulation policy of a price-taking and risk neutral firm operating in the presence of decreasing returns to scale. We extend these previous studies in two ways, by assuming that the underlying price dynamics follows a one-dimensional but otherwise general linear diffusion process and that the production technology is characterized by a differentiable but otherwise general production function subject to decreasing returns to scale. In this way our analysis covers a broad class of descriptions both for the production function and for the underlying stochastic price dynamics which, within our approach, includes most typically applied mean reverting models as well. This is advantageous, since it admits the analysis of the general properties of optimal accumulation policies within this class of investment problems and characterizes those circumstances under which the results obtained by relying on explicitly parameterized models based on exponentially growing prices remain qualitatively valid.

Instead of tackling the stochastic capital accumulation problem directly, we follow the seminal studies by Pindyck (1988) and Bertola (1998) and focus on the decision to acquire or sell a marginal unit of capacity. In this manner, the original accumulation problem is transformed into a simpler timing problem characterizing the marginal value of capital as the value of a single discrete investment opportunity that depends on the prevailing productive capacity, but is independent of the subsequent decisions to either invest or disinvest. We express the value of a marginal unit of capacity in terms of a decomposition capturing the values of its option components and, for the sake of comparison, separately analyze the two associated optimal accumulation problems subject to either irreversible investment or disinvestment. We characterize the marginal value of capital explicitly and demonstrate that the optimal accumulation policy can be characterized in terms of two boundaries at which the productive stock is optimally adjusted. According to the optimal accumulation rule, a further unit of capacity should be acquired whenever the expected cumulative present value of the revenue product it generates exceeds a critical threshold at which the value of a marginal unit of capacity coincides with its acquisition cost. Analogously, disinvestment is optimal along the optimal accumulation path as soon as the expected cumulative present value of the revenue product of capital falls below a critical threshold at which the marginal value of capital coincides with its selling price. Since each unit of capacity decreases the marginal return it generates, we found that the optimal accumulation rule can also be interpreted as a requirement that incremental adjustments to capacity are made each time the underlying price hits one of the optimal boundaries. Whenever the underlying price is between these two critical boundaries the productive capacity is maintained unchanged and the firm continues production with its existing stock. Consequently, the optimal accumulation path consists typically of potentially long periods of time where the firm operates with the prevailing capacity, after which the firm enters into regimes of either very intense investing or disinvesting. The duration of these periods depends on the expected growth rate of the underlying price as well as on its volatility. An interesting property of the optimal incremental accumulation policy is that it is path dependent and, therefore, the future optimal capacities are not only sensitive with respect to changes in the initial stock and price, they are also profoundly affected by the evolution of the underlying price dynamics—especially its historical extreme values (cf. Dixit, 1992). We also characterize the value of the optimal accumulation policy and prove that it can be expressed in terms of the values of the three available operational modes of the firm: investing, operating without changing capacity, or disinvesting. As usually in microeconomic theory, our findings show that an irreversible decision should be exercised whenever the marginal revenues it generates coincide with its marginal costs.

When investment is completely reversible, the optimal myopic accumulation rule dictates that the firm should maintain its productive stock at a level where the expected cumulative present value of the marginal revenue product of capital coincides with its acquisition cost. The inability to sell excess capital at its acquisition price results in a situation where this principle is no longer valid, since a rational forward-looking firm has to take into account that the acquisition costs of a marginal unit are partly sunk and cannot be recovered by selling the marginal unit, if the investment decision turns out to be poor afterwards. Our analysis supports this view and demonstrates that the revenue product of a marginal unit of capacity at the optimal investment boundary exceeds the interest on the marginal acquisition cost (i.e., the marginal user cost of investing). Similarly, our analysis shows that the interest on the selling price of a marginal unit of capacity at the optimal disinvestment boundary exceeds the revenue product it would generate if it were not sold. Therefore, our findings extend the observations made originally by Dixit (1989, 1992) and demonstrate that hysteresis prevails within our setting as well (for associated results on hysteresis, see also Abel and Eberly, 1999; Dias and Shackleton, 2011). It is, however, worth emphasizing that costly reversibility is shown to reduce