Journal of Economic Behavior & Organization
Vol. 57 (2005) 408–420

JOURNAL OF
Economic Behavior
& Organization

www.elsevier.com/locate/econbase

Capital accumulation games with a non-concave production function

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Available online 13 June 2005

Abstract

We consider an economy with a single capital stock that two agents strategically exploit by choosing a consumption profile over an infinite time horizon. We analyze two different games and their corresponding equilibria. In one game firms are able to pre-commit and choose simple time functions as their strategies. In the other game agents are assumed to employ Markov strategies. It turns out that in the case of pre-commitment there exists a threshold level of the capital stock such that if the initial stock is above this threshold, equilibrium consumption converges to the efficient steady state while if the initial condition is below it, the capital stock converges to zero. In case of the Markov equilibrium there exists a unique interior steady state that is globally stable.

JEL classification: C73; D91; C62

Keywords: Non-concave production function; Capital accumulation; Open-loop Nash equilibrium; Markov perfect equilibrium

1. Introduction

Traditionally optimal growth theory is concerned with the derivation of an optimal consumption (or savings) plan of a representative agent when the production function in the

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0167-2681/$ – see front matter
doi:10.1016/j.jebo.2005.04.004

economy is concave and preferences of the consumer exhibit decreasing marginal utility. In such an economy (in particular in the case of a strictly concave production function) there exists a unique long-run steady state capital stock that is globally stable (i.e. the optimal capital stock converges to the steady state level independent of the initial condition).

While the single consumer optimal growth model is a standard vehicle of economic theory, interest recently moved towards models with multiple agents that strategically consume (i.e. the focus in this literature has shifted from representative consumer models to dynamic game models). Examples for dynamic games with capital accumulation are Dutta and Sundaram (1993), Dockner and Sorger (1996), and Dockner and Nishimura (2001, 2004). Although many of those models are interpreted as renewable resource models, they essentially have the features of optimal growth models with multiple agents and strategic consumption of a single resource.

There are several distinguishing features of strategic growth models. First, agents in the growth game strategically interact, and hence the consumption profile of one depends on the consumption of the other player. Second, agents can choose their consumption strategies according to different strategy spaces so that eventually they are playing different games. One popular assumption is to concentrate on the open-loop game. In this setting each player chooses his consumption strategy as a simple time path at the beginning of the game and commits himself to stick to these preannounced paths for the entire duration of the game. This requires that agents have the ability to commit, a rather restrictive assumption in dynamic game theory.

As an alternative, agents can play a game without commitment and use Markov strategies instead. In this case each player chooses a decision rule that relates current consumption to the current capital stock in the economy. This behavior results in a subgame perfect Nash equilibrium.

For both types of games, research in strategic growth models has concentrated on the issue of efficiency of equilibria (i.e., does a game equilibrium result in first best allocations of resources), and on the stability and other qualitative properties of equilibrium paths.

In this paper we formulate a standard growth game in which two agents strategically consume a single resource that is produced with a non-concave production function. Our interest focuses on the qualitative properties of open-loop and Markov perfect equilibria. In particular, we concentrate on both the stability and efficiency of strategic consumption paths and relate the results to the standard optimal growth problem with a non-concave production function.

Skiba (1978) was among the first to analyze a continuous time optimal growth model with a convex–concave production function. Davidson and Harris (1981) formulate an investment problem where they deal with stock and flow non-concavities and are able to characterize the optimal investment program. More recent contributions to optimal growth models with non-concave production include the papers by Long et al. (1997), Askenazy and Le Van (1999).

A complete characterization of an optimal program for a general discrete time growth model with non-concave production can be found in Dechert and Nishimura (1983).[2] The intuition of their results can be carried over to the continuous time case.

[2] See also Majumdar and Nermuth (1982) and Mitra and Ray (1984) for similar analyzes.