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ScienceDirect
Journal of Economic Theory 137 (2007) 568–579
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

On the interaction between risk sharing and capital accumulation in a stochastic OLG model with production

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Received 29 October 2003; final version received 12 January 2007
Available online 27 March 2007

Abstract

We analyze the interaction between risk sharing and capital accumulation in a stochastic OLG model with production. We give a complete characterization of interim Pareto optimal competitive equilibrium allocations. Furthermore, we provide tests of Pareto optimality/suboptimality based on (risky) rates of return only.

JEL classification: D61; D91; E13; E43

Keywords: Stochastic OLG model; Risk sharing; Dynamic efficiency; Pareto optimality

1. Introduction

In this paper, we analyze the interaction between optimal intergenerational exchange under uncertainty ("risk sharing") and capital accumulation in a stochastic OLG model with production. Our contribution is twofold. First, we characterize Pareto optimality with production. Second, we examine the relation to dynamic efficiency and deal with observational implications of theoretical conditions of (sub-)optimality. As is well known, in pure exchange OLG economies the first welfare theorem fails to hold, i.e. competitive equilibria may fail to be Pareto optimal [1,21].

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doi:10.1016/j.jet.2007.01.012

A characterization of Pareto optimality in a pure exchange OLG model under certainty was given by Balasko and Shell [2] and Okuno and Zilcha [18]. Under uncertainty, a characterization of (interim) Pareto optimality in a pure exchange OLG model was derived by Chattopadhyay and Gottardi [9].

Another strand of literature in capital theory with a long-standing tradition is concerned with an a priori stronger form of inefficiency that is independent of specific household preferences, namely the literature on the possibility of capital overaccumulation (dynamic inefficiency) (see [3,4] for a good discussion). Of course, saving too much capital in a competitive equilibrium in the sense that an alternative capital path can uniformly increase aggregate consumption implies Pareto suboptimality. This literature was initiated by Malinvaud [15] and then extended to growth models by Phelps [20] and Diamond [13] and more general infinite-horizon production problems by Cass [7]. [1] An extension of the dynamic efficiency issue under certainty to a setting with uncertainty has been given by Zilcha [23] and Dechert and Yamamoto [10].

Characterizations in terms of prices in competitive equilibrium of these two notions of efficiency—although the notions themselves are conceptually distinct—are based on the same formal argument, so that the conditions in terms of competitive equilibrium prices for dynamic inefficiency and Pareto optimality in pure exchange models coincide under certainty (see [7,18] or [2]). Under uncertainty, risk sharing issues make important adaptations and qualifications necessary. Chattopadhyay and Gottardi [9] extend and generalize a Cass-type argument to a stochastic pure exchange model.

Our first contribution (Proposition 1) is a complete characterization of interim Pareto optimality in a stochastic Diamond model, extending the proof by Chattopadhyay and Gottardi [9] to a setup with production. [2] It turns out that in a competitive equilibrium the characterization of interim Pareto optimality is equivalent to the one in a pure exchange setup. This holds although in a model with production there are more feasible deviations from the competitive equilibrium than under pure exchange and it is not a priori possible to restrict attention to pure transfer schemes.

As a special case of our optimality characterization we obtain Zilcha's [23] dynamic efficiency characterization (Proposition 2). Contrary to the case of certainty, the conditions for dynamic efficiency and Pareto optimality do not coincide under uncertainty. This means that the possibility of overaccumulation of capital is not necessarily related to what Manuelli [16] called the "intergenerational risk sharing" part of the efficiency problem. Nevertheless, we show that there is a relationship between the risky rate of return on capital—and more generally the rate of return of some arbitrary asset—and interim Pareto optimality of competitive equilibria (Proposition 3). Using this relation, we derive a sufficient condition for interim Pareto optimality that requires only the knowledge of the rates of return of some arbitrary asset for each date-event and not the full set of contingent claims prices. This set of results is related to recent independent research by Chattopadhyay [8] but is more general (see the discussion after Proposition 4).

The paper is organized as follows. Section 2 describes the stochastic Diamond model. Section 3 gives a complete characterization of interim Pareto optimality in a competitive equilibrium. Section 4 reconsiders the condition for dynamic efficiency and derives optimality tests based on risky and other rates of return. All proofs are given in the appendix.

[1] Tirole [22] analyzed the relationship between dynamic efficiency and the existence of bubbles as well as the Pareto optimality of bubbly equilibria.

[2] Demange and Laroque [12] derive a partial classification (but not a characterization) of interim Pareto optimality in a stochastic OLG model with production, however, under relatively strong stationarity assumptions.