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A DNS-curve in a two-state capital accumulation model: a numerical analysis

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Abstract

In this paper we study a capital accumulation model in an optimal control theoretic framework, where the capital stock and the investment rate are modelled as state variables and the change in the investment rate as control. Adjustment costs are introduced for investment rate and its change. Moreover, we model network externalities by a convex segment in the revenue function, which implies the existence of two long-run optimal steady-states, one with a low level and the other with a high level capital stock. It depends on the initial capital endowment and initial investment rate to which steady-state it is optimal to converge. We numerically compute a curve in the state plane, for which it holds that, when starting from a point on this curve, the decision maker is indifferent between going to either one of these steady-states, and identify this curve as the DNS-curve. The negative slope of the DNS-curve indicates that there is a trade-off between the initial capital endowment and initial investment rate.

JEL classification: C61; C62; D92

Keywords: DNS-curve; Capital accumulation; Optimal control; Numerical methods; Multiple steady-states

1. Introduction

Dechert and Nishimura (1983) analyze a discrete time Ramsey model in which the production function is convex–concave. They show that, provided that the interest rate has an intermediate value, the optimal path converges to a steady-state only if the initial

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capital stock is above a critical value, otherwise it converges to zero. The first time this phenomenon appears in the literature is Skiba (1978), who studies a one-dimensional continuous time growth model with a non-convex technology. Dechert (1983) and Davidson and Harris (1981) analyze a firm capital accumulation model, where the revenue function contains a convex segment. They show that under a particular scenario a threshold value of the initial capital stock exists, above which it is optimal to converge to the larger saddle point (i.e. the saddle point with the larger capital stock level) and below which convergence to the smaller saddle point is preferable. Honoring these contributions we want to call such a threshold as the DNS-(Dechert–Nishimura–Skiba–) point. We define a DNS-point as a point at which there are two optimal paths, where each of them approaches a different steady-state. The decision maker is indifferent concerning the choice of these two paths.

All these papers have in common the fact that the optimal paths are history dependent caused by (local) non-convexities. An unstable steady-state is crucial for determining the threshold separating the starting points of the optimal trajectories leading to the different long run outcomes. In a recent paper by Wirl and Feichtinger (1999) it is found, however, that (local) non-convexities are by no means necessary for the occurrence of such thresholds. They provide two mechanisms, viz, growth and control state interactions, which can lead to history dependence in a strictly concave framework.

All the above-mentioned contributions consider a dynamic optimization problem with a one-dimensional structure. Intuition suggests that extending this feature to the class of optimal control models with two state variables will lead to the occurrence of a set of DNS-points, which we will call DNS-set. In the literature contributions that deal with this topic are scarce. Brock and Dechert (1983) prove the existence of a DNS-set, but the exact location of it is not determined. To best authors' knowledge no paper has appeared in which the shape of a DNS-set is worked out. This is maybe caused by the fact that finding a DNS-set analytically is not an easy task. In this paper we determine a DNS-set in a two-dimensional capital accumulation model by computing numerically the optimal trajectories.

The capital accumulation framework has been extensively analyzed in the literature assuming a constant or decreasing returns to scale technology and adjustment costs of investment (Eisner and Strotz, 1963; Lucas, 1967; Gould, 1968). One specific feature of our model is that, besides the prevalent adjustment costs associated with investment, additional costs are incurred for changes in the investment rate. Another specific feature in our model is that our formulation admits the existence of network externalities. By a network externality it is meant that the value of a good increases with the number of users (see Economides, 1996). An example would be that it is beneficial to the users of a certain software package, if this package is used by many other people. Similarly, for the user of some GSM network the value increases with the number of users because of the significantly reduced rates within the network. In the model this is reflected by an inverse demand function, which increases for intermediate quantities produced. This implies that the firm's revenue function is convex in a segment of intermediate capital stock values, whereas the firm's revenue function is concave for small and large capital stock values outside this segment. Similarly, in Davidson and Harris (1981) and Dechert (1983) a revenue function occurs, which contains one convex segment.