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On the existence of Nash equilibria in an asymmetric tax competition game[☆]

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ABSTRACT

In this methodological paper, we prove that the famous tax competition game introduced by Zodrow and Mieszkowski (1986) and Wildasin (1988) in which the capital is completely owned by foreigners possesses a Nash equilibrium even when the assumption of symmetric jurisdictions is dropped. The normality of both private and public goods is all that is needed concerning restrictions on preferences when a peculiar regime of taxation is ruled out. Moreover, we show that conditions about technology allowing for the existence of a Nash equilibrium are satisfied by most of the widely-used production functions.

1. Introduction

In the established literature on tax competition the existence of a Nash equilibrium is assumed (see Zodrow and Mieszkowski, 1986; Wilson, 1985, 1986 and Wildasin, 1988). These studies focus on the comparative statics of Nash equilibria, and demonstrate that public services are provided at inefficiently low levels in equilibrium. However, little attention has been devoted to the question of whether such equilibria do exist. This is for the most part because the demonstration is very difficult, as noted by Laussel and Le Breton (1998): “Both the existence and uniqueness issues are difficult in general and have not been up to now dealt with in the literature. It seems however of primary interest to solve them in order to understand the comparative statics of the equilibrium.”

Some results have already been established in the literature for particular cases. Firstly, Bucovetsky (1991) demonstrated the existence of a Nash equilibrium in tax rates in the case of two regions and quadratic production functions. A second and important result was highlighted by Laussel and Le Breton (1998) who proved the existence of a symmetric Nash equilibrium when private and public goods are perfect substitutes and when capital is not owned by residents. In addition, this framework enables the authors to prove the uniqueness of the equilibrium, which is the primary purpose of their paper. In a more recent paper, Bayindir-Upmann and Ziad (2005) apply a weaker concept than the standard Nash equilibrium – the concept of a second-order locally consistent equilibrium (2-LCE) – which is a local Nash equilibrium (i.e., a small deviation is undesirable). With this tool, the authors are able to show both the existence and uniqueness of a symmetric equilibrium in tax rates when regions are homogeneous and when either (i) there are only two regions, (ii) capital demand curves are convex, or firms apply (iii) CES, (iv) Cobb–Douglas, or (v) logistic production functions. More recently, Dhillon et al. (2007) investigate the existence of a Nash equilibrium in a symmetric tax competition model where the public good enters the production function. Rothstein (2007) analyses the fiscal competition game as a game with discontinuous payoff and demonstrates the existence of a pure strategy Nash equilibrium for this kind of game under several assumptions respecting the production function. Rothstein moves away from the standard fiscal competition game “à la Wildasin” by assuming: first, an ad valorem tax; and second, that the aggregate amount of mobile capital is fixed in all regions. Finally, Petchey and Shapiro (2009) examine the problem of the existence of a Nash equilibrium in a tax competition model when governments are no longer benevolent but only make constrained efficient choices.

A key point of this paper is that we deal with asymmetric regions. The literature on asymmetric tax competition is mainly based on two articles by Wilson (1991) and Bucovetsky (1991). Both assume that regions differ in their population and show that the “small” region

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may benefit from the tax competition by attracting capital from the "large" region thanks to the tax competition mechanism. In the present paper, we retain the methodological question regarding the existence of a Nash equilibrium in the tax competition model "à la Wildasin". We depart from this seminal model by assuming that capital is owned by foreigners and we extend it to regions with different production functions.[1] In doing so, we extend the existing literature by proving the existence of a Nash equilibrium in a more general framework. Our paper is in line with Laussel and Le Breton (1998) and Bayindir-Upmann and Ziad (2005), but we basically depart from their analysis by relaxing the assumption of symmetry. We also depart from the paper by Rothstein, firstly by considering proportional taxes, whereas Rothstein uses an ad valorem tax; secondly, by establishing a weaker condition of existence than the quasiconcavity condition of Rothstein, and thirdly, by directly deriving our result in the tax competition model. To prove our results, we use several assumptions: that goods are normal; that the demand for capital is convex; and that elasticity of the demand for capital is non increasing in absolute value. Moreover, we use an additional condition on preferences which enables us to rule out the particular regime of taxation in which the return on capital is zero and capital owners limit their capital supply.

This paper is organized as follows. The second section outlines the tax competition model for a given number of regions, notation and description of the model being taken for the most part from Bayindir-Upmann and Ziad (2005). In Section 3 we establish a general theorem for the existence of the Nash equilibrium in fiscal competition. In Section 4 we introduce several examples of the commonly-used production functions and check the tractability of the result in Section 3. The final section summarizes our conclusions.

2. The model

Consider n $(n\geq 2)$ jurisdictions inhabited by a given number of homogeneous residents that we normalize to one without loss of generality. A fixed number of competitive firms produce a homogeneous output in each jurisdiction using capital and some fixed factor(s) (land or labour). Aggregating production over all firms in each region allows us to treat the industry of one jurisdiction as one competitive firm. Let f^i be the production function of the firm in jurisdiction i, f^i is assumed to be monotonically increasing and strictly concave in capital, K_i. Fixed factors as explicit arguments of f^i are suppressed so that the production function is expressed in terms of capital only. The jurisdiction i firm's profit can be written as

$$\Pi_i = f^i(K_i) - p_i K_i, \qquad i = 1, \ldots, n \tag{1}$$

where p_i denotes the after-tax price of capital in jurisdiction i. Equating the price of capital to the value of its marginal product, $f_k^i(K_i) = p_i$ determines region i's capital demand as a function of the corresponding after-tax price of capital, $\hat{K}_i(p_i)$.

Let $U(X_i,P_i)$ be the utility that the representative household of jurisdiction i derives from the provision of the public good, P_i, and from the consumption of the private good, X_i, produced by the firms. The utility function $U(X_i,P_i)$ is twice-continuously differentiable and monotonically increasing. The source of the household's income is twofold: one part from the provision of the fixed factor, which is exclusively owned by local residents; and one part from their initial capital endowment. Let $\theta_i \in [0,1]$ denote region i's share of the fixed national capital stock $\overline{K}$, and p_i, the net return of capital in region i. The private budget constraint of the consumer in region i amounts to

$$X_i = f^i(K_i) - p_i K_i + \theta_i \rho_i \overline{K} \tag{2}$$

for each jurisdiction $i = 1, \ldots, n$.

In the following of the paper, we assume $\theta_i = 0\,\forall i$. One explanation is that capital is owned by agents outside the jurisdictions under consideration, as in Wildasin (1988), Laussel and Le Breton (1998) and Rothstein (2007).[2] Another explanation is to consider that the income of the resident capital owners does not affect the utility of the representative agent in each jurisdiction.[3]

Each local government provides a public good that it finances by taxing the mobile capital at a tax rate $t_i \geq 0$.[4] The budget constraint of jurisdiction i is given by

$$P_i = t_i K_i. \tag{3}$$

When choosing the level of the tax rate, each local government acts as a benevolent one and aims to maximize its representative resident's utility. In doing so, each local authority behaves non-cooperatively and treats its specific tax on capital t_i, $i = 1, \ldots, n$, as the strategic variable. This leads to a tax competition game between jurisdictions.

The capital market clearing condition implies that aggregate demand for capital must equal capital supply:

$$\sum_{i=1}^{n} K_i = \overline{K}, \tag{4}$$

for some exogenously given capital supply $\overline{K}$.

Capital being freely mobile across regions, the arbitrage condition equals the net return of capital in each jurisdiction:

$$\rho = f_k^i(K_i) - t_i (= \rho_i), \qquad \forall i = 1, \ldots, n. \tag{5}$$

Let $\mathbf{t} = (t_1, \ldots, t_n)$ be the profile of tax rates, $\mathbf{t}_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$ be the profile of all tax rates except t_i; whereas $(t_i, \mathbf{t}_{-i})$ stands for $(t_1, \ldots, t_{i-1}, t_i, t_{i+1}, \ldots, t_n)$. Both Eqs. (4) and (5) define the equilibrium allocation of capital and the equilibrium of the net return of capital, i.e. $K_1(\mathbf{t}), \ldots, K_n(\mathbf{t})$ and $\rho(\mathbf{t})$.

Capital supply $\overline{K}$ is assumed exogenously fixed, whenever the solution of Eqs. (4) and (5) requires that capital earns a non-negative net return, $\rho \geq 0$. When $\rho < 0$, we suppose that capital owners limit capital supply. The resulting allocation of capital is a vector $(K_1, K_2, \ldots, K_n)$ with $\sum_{i=1}^{n} K_i = K^* < \overline{K}$, for which $f_k^i(K_i) = t_i$.

We assume that if for some profile $\mathbf{t} = (t_1, t_2, \ldots, t_n)$ we have $\rho(\mathbf{t}) = \rho_i(\mathbf{t}) = 0$ for each i, and if a region j raises its tax rate from t_j to $t_j + \Delta t_j$ such that $\rho_j(t_{-j}, t_j + \Delta t_j) < 0$ while the other regions keep their tax rate unchanged, then capital owners limit supply only in region j until $\rho_j = 0$.

As a result, as in Bucovetsky (1991), we have three regimes:

1. Positive return regime: $\sum_{i=1}^{n} K_i = \overline{K}, \rho > 0$,
2. Excess supply regime: $\sum_{i=1}^{n} K_i = K^* < \overline{K}, \rho = 0$,
3. Boundary between regimes: $\sum_{i=1}^{n} K_i = \overline{K}, \rho = 0$.

It is shown in Bucovetsky (1991) that the derivative $\frac{\partial K_i}{\partial t_i}$ changes discontinuously at any tax vector $(t_1, t_2, \ldots, t_n)$ at which $\rho = 0$ and $\sum_{i=1}^{n} K_i = \overline{K}$, i.e. in the boundary between regimes. In any regime

[1] In his paper, Wildasin (1988) uses also the assumption of absentee owners of capital in the jurisdictions to derive his results in the special case of identical jurisdictions.

[2] Alternatively, Zodrow and Mieszkowski (1986), Bucovetsky (1991) or Bayindir-Upmann and Ziad (2005) assumed that owners of capital own an equal share of the total capital stock.

[3] This could be the case if we consider two groups of agents in each jurisdiction, one which does not own capital but controls decision making, say the "poor/middle class", and one which owns capital and passively consumes its net income of capital, say the "rich class". This comment has been suggested by one of the referees.

[4] Capital subsidies cannot be funded in this model since there are no other taxes available.