Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Regional Science and Urban Economics 35 (2005) 1–22
regional SCIENCE & urban ECONOMICS
www.elsevier.com/locate/econbase

Existence of equilibria in a basic tax-competition model

Thorsten Bayindir-Upmann[a,*], Abderrahmanne Ziad[b]

[a] *Institute of Mathematical Economics, University of Bielefeld, P.O. Box 100131, 33501, Bielefeld, Germany*
[b] *Groupement Economie Mathématiques, et de Micro-Economie Appliquée, University of Caen, Esplanade de la Paix, Caen 14000, France*

Received 28 March 2002; accepted 29 January 2003
Available online 10 December 2003

Abstract

While the properties of equilibria in games of interjurisdictional tax competition have been heavily scrutinized, no hard evidence is available that these equilibria exist. Even for the fundamental tax competition game introduced by Zodrow and Mieszkowski [J. Urban Econ. 19 (1986) 356] and Wildasin [J. Public Econ. 35 (1988) 229], only few positive, but arguably restrictive, results have been derived. Applying a weaker concept than the standard Nash equilibrium—the concept of a second-order locally consistent equilibrium (2-LCE)—we are able to show both the existence and uniqueness of a symmetric equilibrium in tax rates, when regions are homogeneous and either (i) there are only two regions, (ii) capital demand curves are concave, (iii) the inverse of the elasticity of the marginal product of capital is not increasing, or firms apply (iv) CES, (v) Cobb–Douglas, or (vi) logistic production functions.

JEL classification: C72; H21; H42; R50
Keywords: Second-order locally consistent equilibrium; Nash equilibrium; Tax competition; Existence and uniqueness of equilibria

1. Introduction

Since the seminal papers of Zodrow and Mieszkowski (1986) and Wilson (1985, 1986), much effort has been spent in order to investigate the properties of equilibria in games of interjurisdictional competition. Presumably the most substantial part of the literature

* Corresponding author. Tel.: +49-521-106-4898; fax: +49-521-106-2997.
E-mail address: TUpmann@wiwi.uni-bielefeld.de (T. Bayindir-Upmann).

0166-0462/$ - see front matter
doi:10.1016/j.regsciurbeco.2003.01.001

focuses on tax competition: a noncooperative game where local governments providing (local) public goods compete for interregionally mobile capital by means of capital taxation. Starting from the classical result that public services are provided at inefficient low levels in equilibrium, many interesting variants and extensions of the basic model have been scrutinized and various interesting results have been derived in the sequel (which we are unable to review here).

While the properties of the resulting equilibria have been explored heavily, little attention has been devoted to the question whether these equilibria actually exist. Yet, as long as we are not able to demonstrate the existence of such an equilibrium—let alone its uniqueness—one cannot be sure that we are not discussing the properties of an empty set. Nevertheless, most authors just *assume* the existence of such an equilibrium, and only few have considered the existence problem, though. A notable contribution is, for example, the paper of Bucovetsky (1991). He demonstrates the existence of a Nash equilibrium in tax rates for the case of two regions and quadratic production functions. A more recent attempt to tackle the existence problem in the two-country case has been undertaken by Laussel and LeBreton (1998). They prove the existence and uniqueness of a symmetric Nash equilibrium. Unfortunately, their result is obtained under rather demanding assumptions. The assumption of absent capital owners raises the question as to whom the capital income accrues. As Laussel and Le Breton argue, this either reduces the model to a partial equilibrium framework or requires a tax authority which ignores the welfare of capital owners, a feature which may result in the framework of a political economy when the majority voter does not possess any capital. Arguably, even more restrictive is the assumption of linearity of each region's welfare function in both public and private consumption. As Laussel and Le Breton acknowledge, this assumption deprives the model of its economic relevance: It completely removes the problem of inefficient provision of public goods—and even the necessity of providing public goods at all. Indeed, as these authors argue, 'solving the existence question when both aspects [the competition and the provision aspect] are present looks rather difficult' (p. 285). This is actually true—and is exactly the topic of this paper. We demonstrate both the existence and uniqueness of a symmetric equilibrium in a standard tax competition model by application of a slightly weaker equilibrium concept—but without removing the economic essentials of the model.

The standard equilibrium concept in models of fiscal competition is that of a Nash equilibrium. Yet, as the literature demonstrates, the existence and uniqueness issue is hard to deal with under the notion of a Nash equilibrium. This great difficulty is due to the fact that the welfare (or payoff) functions are derived functions, as they are defined in terms of tax rates and not in terms of the underlying real variables—the consumption levels. This requires that the properties of these indirect functions must be derived from both the original utility and the production function. As a consequence, the derived welfare functions are seemingly not (quasi-)concave on their domain nor is the game supermodular.

In order to deal with this difficulty, the concept of the kth-order locally consistent equilibrium (k-LCE) has been introduced. Assuming that each player chooses his/her strategy variable according to a payoff function defined over the whole set of strategies, a k-LCE is a strategy profile at which players correctly perceive the kth-order Taylor expansion of their payoff function. For example, a first-order locally consistent equilibrium (1-LCE) of a game is a configuration of strategies at which the first-order condition