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Power laws in firm size and openness to trade: Measurement and implications ☆

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ABSTRACT

Existing estimates of power laws in firm size typically ignore the impact of international trade. Using a simple theoretical framework, we show that international trade systematically affects the distribution of firm size: the power law exponent among exporting firms should be strictly lower in absolute value than the power law exponent among non-exporting firms. We use a dataset of French firms to demonstrate that this prediction is strongly supported by the data, both for the economy as a whole and at the industry level. Furthermore, the differences between power law coefficients for exporters and non-exporters are larger in sectors that are more open to trade. While estimates of power law exponents have been used to pin down parameters in theoretical and quantitative models, our analysis implies that the existing estimates are systematically lower than the true values. We propose two simple ways of estimating power law parameters that take explicit account of exporting behavior.

1. Introduction

Many relationships in economics appear to be governed by power laws. A distributional power law is a relationship of the type: $Pr(X>x) = Cx^{-\zeta}$ where $Pr(X>x)$ is the probability that a random variable X is greater than x, and C and ζ are constants. Power laws arise in a variety of contexts, such as the distribution city size (Zipf, 1949), income (Champernowne, 1953), firm size (Axtell, 2001), and sectoral trade flows (Hinloopen and van Marrewijk, 2006; Easterly et al., 2009).

The literature has emphasized the importance of the precise value of the power law exponent, ζ. For instance, for the distribution of firm size in the U.S., Axtell, (2001) reports a range of estimates between 0.996 and 1.059, very precisely estimated with standard errors between 0.054 and 0.064. The literature has sought to both explain why ζ is close to 1 (a phenomenon known as Zipf's Law) and to explore its implications in a variety of contexts. It has been argued that Zipf's Law will arise when the variable of interest, be it city, or firm size, follows a geometric Brownian motion (Gabaix, 1999; Luttmer, 2007; Rossi-Hansberg and Wright, 2007). At the same time, the precise magnitude of the power law exponent has been shown to matter for such different phenomena as macroeconomic fluctuations (Gabaix, 2011; di Giovanni and Levchenko, 2010a), regulation of entry (di Giovanni and Levchenko, 2010b), and executive compensation (Gabaix and Landier, 2007).

This paper revisits the power law in the distribution of firm size in the context of international trade. We first set up a simple version of the Melitz (2003) model of production and trade, adopting the common assumption that the distribution of firm productivities is Pareto. This model is naturally suited to studying the firm size distribution because of its emphasis on heterogeneous firms. The Melitz–Pareto framework delivers a power law in firm size. However, it also predicts that in the presence of international trade, the power law exponent in the distribution of firm size is not constant. Because larger firms are more likely to export, and the more productive the firm, the more markets it serves, we would expect the estimated power law exponent to be lower in absolute value among exporting firms compared to the non-exporting ones. In other words, in the presence of international trade, power law estimates that do not take

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[1] See Gabaix (2009) for a recent survey.

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into account international trade could be misleading regarding the deep parameters of the economy.[2]

We evaluate these predictions of the Melitz–Pareto model using the data on production and exports for a large sample of French firms. In the full sample that includes all firms, the power law in firm size is strikingly similar to what Axtell (2001) found for the census of U.S. firms. The estimated power law exponent and the fit of the relationship are both nearly identical. However, when we separate the firms into exporting and non-exporting ones, it turns out that in the exporting sample, the power law coefficient is consistently lower, while in the non-exporting sample, consistently higher than in the full sample of firms. This difference is present across all estimators, and is highly statistically significant. In addition, we show that, as predicted by theory, the power law exponent for exporting firms converges to the power law exponent for domestic firms as we restrict the sample to larger and larger exporters.

Results based on the economy as a whole are not conclusive evidence that exporting behavior *per se* generates the difference in power law coefficients between exporters and non-exporters. For instance, it is well known that exporters tend to be larger than non-exporters, and thus the findings could be driven by a departure from an exact power law for larger firms. We address this concern in two ways. First, to focus specifically on the role of international trade, we estimate power law coefficients for exporting and non-exporting firms by sector. At the disaggregated industry level, the estimates exhibit the same pattern: power law coefficients for exporters are systematically lower than for non-exporters. More tellingly, the differences between power law coefficients are larger in industries that are more open to trade, a striking regularity consistent with the theoretical intuition developed in the paper. Second, we re-estimate the power laws on sub-samples of exporters and non-exporters of the same mean size, confirming the main result. All of these pieces of evidence lend empirical support to the main idea of the paper: international trade systematically changes the distribution of firm size, and inference that does not take that into account will likely lead to biased estimates.

One of the reasons empirical power law estimates are important is that they are used to pin down crucial parameters in calibrated heterogeneous firm models (see, among many others, Helpman et al., 2004; Chaney, 2008; Buera and Shin, 2010; di Giovanni and Levchenko, 2010a). At the same time, quantitative results often depend very sharply on the precise parameter values that govern the distribution of firm size. di Giovanni and Levchenko (2010b) show that welfare gains from reductions in fixed costs are an order of magnitude lower, and gains from reductions in variable costs an order of magnitude higher in a model calibrated to Zipf's Law compared to the counterfactual case in which $\zeta=2$ instead. We return to the Melitz–Pareto model, and propose two alternative ways of estimating the power law parameters that are internally consistent with the canonical heterogeneous firm model of trade. The first is to use a sample of only non-exporting firms. The second is to use only domestic sales to estimate the power law parameter.

We are not the first to provide parameter estimates for the firm size distribution that explicitly account for international trade. Eaton et al. (forthcoming) set up a multi-country heterogeneous firms model and estimate a set of model parameters with Simulated Method of Moments using the data on French firms.[3] The advantage of our approach is simplicity. The alternative estimation strategies proposed here are very easy to implement and do not require any additional modeling or estimation techniques. All they rely on is an appropriate modification of the sample or variables used in estimation. Our approach thus substantially lowers the barriers to obtaining reliable power law estimates, and can be applied easily in many contexts.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 describes the dataset used in the analysis and the methodology for estimating power laws. Section 4 describes the results. Section 5 concludes.

2. Theoretical framework

This section describes the firm size distribution in the canonical heterogeneous firms model, and in particular how it is affected by international trade. Throughout this section, sales are the measure of firm size, though all the empirical results are reported below for both sales and employment as an alternative size variable. Firm sales, S_i, in the economy are said to follow a power law if their distribution is described by[4]:

$$Pr(S_i > s) = Cs^{-\zeta}. \tag{1}$$

The canonical monopolistic competition model with CES demand and heterogeneous firms implies that domestic sales of firm i in market n are given by

$$D_i = M_n \times B_i,$$

where M_n is a measure of the size of domestic demand, which is the same for all firms, and $B_i \equiv a_i^{1-\varepsilon}$ is the firm-specific (but not market-specific) productivity-cum-sales term. In this expression, a_i is the marginal cost of firm i, and ε is the elasticity of substitution between CES varieties.[5]

We postulate that B_i follows a Pareto distribution with exponent ζ. Under some conditions (e.g., Gabaix, 1999; Luttmer, 2007), B_i comes from a random growth model, which yields a value of ζ close to 1. It turns out that in the canonical heterogeneous firms model this is equivalent to assuming that firm productivity is Pareto, but with a different exponent. To see this, suppose that firm productivity has the Pareto cdf: $\Pr(1/a < y) = 1-\left(\frac{b}{y}\right)^{\theta}$. In the autarkic economy, where $S_i = D_i$, the power law follows:

$$Pr(S_i > s) = Pr(M_n B_i > s) = Pr\left(a_i^{1-\varepsilon} > \frac{s}{M_n}\right) = \left(b^{\varepsilon-1}M_n\right)^{\frac{\theta}{\varepsilon-1}} s^{-\frac{\theta}{\varepsilon-1}}, \tag{2}$$

satisfying Eq. (1) for $C = \left(b^{\varepsilon-1}M_n\right)^{\frac{\theta}{\varepsilon-1}}$ and $\zeta = \frac{\theta}{\varepsilon-1}$. The model-implied distribution of sales is depicted in Fig. 1. In addition, this calculation shows that $S_i \sim \text{Pareto}\left(b^{\varepsilon-1}M_n, \frac{\theta}{\varepsilon-1}\right)$.

The result that the power law exponent is constant and equal to $\frac{\theta}{\varepsilon-1}$ holds true in autarky, and also among non-exporting firms in the trade equilibrium. But how does exporting behavior change the firm size distribution? We describe two mechanisms by which exporting tilts the power law relationship systematically to make it flatter (more right-skewed). The first relies on entry into progressively more foreign markets. The second, on stochastic export market entry costs that vary by firm. In the second case, it is possible to obtain a number of analytical results about the distribution of firm sales, and show that it is systematically affected by international trade.

To start exporting from country n to country m, firm i must pay the fixed cost κ_{mni} that potentially varies by firm, and an iceberg per-unit cost of $\tau_{mn} > 1$. It is easy to verify that export sales by firm i to market m can be expressed as $M_m^* B_i$, where M_m^* is m's market size *from the*

[2] This paper focuses on power law estimation because power laws appear to be the best description of observed firm size distributions (Luttmer, 2007). However, the qualitative mechanisms we highlight apply to any other underlying distribution of firm size.

[3] See Arkolakis (2009, 2010) for related theoretical treatments.

[4] Unless otherwise noted, in the discussion below all parameter values are nonnegative.

[5] $M_n = \frac{Y_n}{P_n^{1-\varepsilon}}\left(\frac{\varepsilon}{\varepsilon-1}\omega_n\right)^{1-\varepsilon}$, where Y_n is total expenditure, P_n is the price level, and ω_n is the cost of the input bundle. For details of this type of model, see, e.g., di Giovanni and Levchenko (2010b).