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Measuring the capital charge for operational risk of a bank with the large deviation approach

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ABSTRACT

In this paper, the large deviation approach for computing the capital charge for operational risk of a bank is explored. Firstly, the negatively-associated structure is utilized to measure the dependence between distinct operational loss cells. Secondly, the lower and upper bounds of the tail distribution function of total aggregated loss processes are determined. In addition, first order approximations using a value-at-risk measure are derived. Finally, an important example calculating the capital charge for operational risk under the class of a heavy-tailed distribution is provided.

1. Introduction

Measuring the capital charge for operational risk of a bank has become an important area of research in finance in recent years (e.g., [1–8]). In Refs. [6,7], the Basel committee on banking supervision defines operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. It is especially difficult to measure this new type of risk because operational loss events are extremely rare and produce enormously high losses and because there are relatively few data available on this topic.

One branch of the literature has focused on developing various kinds of risk models for allocating the regulatory capital that covers yearly operational risk exposure with a confidence interval of 99.9%. Several advanced measurement methods have been described, including the internal measurement approach, the scored approach, the loss distribution approach (LDA), and the Bayesian approach (e.g. [1–5,8–29]).

Broadly speaking, measuring the capital charge of a bank is part of risk management. Performance analysis in the banking industry has become a key indicator for assessing the quality of a bank management practices. In Ref. [30], data envelopment analysis (DEA) and neural networks (NNs) were integrated to examine the relative branch efficiency of a large Canadian bank. These authors also provided guidance on how to improve the branch performance and developed a short-term efficiency prediction model. In Ref. [31], the authors developed a new DEA model to assess the dual impacts of the operating and business strategies for the Canadian L&H insurance industry. These authors found that the new DEA model can simultaneously assess the performance of production and investment. Wu [32] developed a new bi-level programming DEA approach for optimizing the performance of decentralized companies by using multiple inputs to produce multiple outputs in a cost-effective manner. In Ref. [33], a predictive scorecard model was introduced to assess account credit worthiness in large banks. These authors pointed out that the internal scorecard was better able to distinguish the 'bads' from the 'goods' than the Bureau Score. Descriptions of other recent advances in risk analysis and management can be found in reports by Wu or Olson [34–43].

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Two definitions for enterprise risk management (ERM) have been developed by COSO and by Wu and Olson [37]. The latter definition provides the most useful way for firms to manage their risks in the sense of an "integrated" risk management. The concrete definition is as follows — "ERM is the integrated process of identification, analysis and either acceptance or mitigation of uncertainty in investment decision making". As pointed out by Valle and Giudici [15], one of the shortcomings of LDA is its lack of qualitative data (such as expert opinions) to define prior probabilities. Scorecards, which were well developed by [37], can be used to further study LDA, and the internal risk rating system and the measurement of the performance of capital charges in multiple risk cells can be developed by validating predictive scorecards.

It is often assumed that the loss frequency process $N_i(t), i = 1, 2, \dots, d$ for every risk cell is a homogeneous Poisson process. The assumption of a constant rate is unrealistic in many practical examples. As reported by Ref. [1], the variance of the monthly frequency series is much higher than the mean rate in some risk cells, suggesting that frequency count data are over-dispersed. Empirical evidence of the over-dispersion phenomena was also documented by Refs. [5,9]. This problem can be dealt with by using a nonnegative integer-based process, also known as the loss frequency process.

Gaining an accurate understanding of the degrees of dependence among losses in various units of measurement is critical to estimating the total capital charge under the LDA. In Ref. [4], it was pointed out that the number of external fraud events has historically been high (or low) when the number of internal fraud events is high (or low). This inter-dependence of risk cells has been convincingly shown to be an aggregated loss correlation; this dependence is elegantly illustrated by the underlying correlation between frequencies but not by correlations in severity in the LDA framework. Other recent advances in describing the dependence of operational risks can be found in Ref. [9]. Moreover, empirical findings show that the correlation between two aggregate losses is generally weak (typically below 5%); as a result, a suitable tool for measuring such a weak dependence is needed. This correlation could be completely dependent, co-monotonic, positively/negatively dependent, independent, etc.

The remainder of this paper is organized as follows. Section 2 outlines the univariate and multivariate LDA models used in this paper and explores concepts related to these models. In Section 3, the lower and upper bounds about the tail distribution function of total aggregated loss processes are provided, and the large deviation results describing first passage time are explored. In Section 4, first order approximations using a value-at-risk measure are derived. In addition, an important example calculating the capital charge for operational risk under the class of a heavy-tailed distribution is provided. Finally in Section 5, the complete proofs of the results are listed.

2. Preliminary results

In this section, we provide information on the univariate and multivariate LDA models used in this paper. It is noteworthy that the loss frequency process $N(t)$ is assumed to be a nonnegative integer-based process instead of a homogeneous Poisson process.

Definition 2.1 (*Univariate Loss Distribution Approach (LDA) Model*).

1. The severity process: the severities are modeled by a sequence of positive independent and identically distributed (i.i.d.) random variables $\{X_k, k \in \mathbb{N}\}$. Let F be the distribution function (df) of X_k with $\mu = EX_1 < \infty$.
2. The frequency process: the random variable $N(t)$ representing losses during the time interval $[0, t]$ is a nonnegative integer-based process; we also assume that $\lambda(t) = EN(t) < \infty$ for any $t > 0$ and $\lambda(t) \to \infty$ as $t \to \infty$.
3. The severity process and the frequency process are assumed to be independent.
4. The aggregated loss process is defined as $S(t) := \sum_{k=1}^{N(t)} X_k$.

Definition 2.2 (*Extended Regular Variation (ERV) Class*). $F \in ERV(-\alpha, -\beta)$ for $1 < \alpha \le \beta < \infty$ if F satisfies this condition, then for any $y > 1$

$$y^{-\beta} \le \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le y^{-\alpha}. \tag{2.1}$$

This is also applicable for any $v > 1$

$$v^{\alpha} \le \liminf_{x\to\infty} \frac{\overline{F}(x/v)}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}(x/v)}{\overline{F}(x)} \le v^{\beta}. \tag{2.2}$$

Definition 2.3 (*Multivariate LDA Model*).

1. Let every ith risk cell be a one-dimensional LDA model with an aggregated loss process S_i, severity tail distribution $\overline{F}_i$, severity expectation μ_i and frequency process $N_i(t)$ with parameters $\lambda_i(t)$.
2. The dependence between the aggregated operational losses $\{S_i, i = 1, 2, \dots, d\}$ of the Basel cells is modeled by a negatively-associated (NA) structure. More precisely, for any disjoint subsets A_1, A_2 of the set $\{1, 2, \dots, d\}$, the following inequality holds:

$$Cov(f_1(S_i, i \in A_1), f_2(S_j, j \in A_2)) \le 0, \tag{2.3}$$