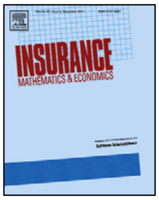
Insurance: Mathematics and Economics 49 (2011) 565–579

Contents lists available at SciVerse ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime



Analytic loss distributional approach models for operational risk from the α-stable doubly stochastic compound processes and implications for capital allocation

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ARTICLE INFO

Article history:
Received March 2011
Received in revised form
August 2011
Accepted 24 August 2011

Keywords:
Operational risk
Loss distributional approach
Doubly stochastic Poisson process
α-Stable
Basel II
Solvency II

ABSTRACT

Under the Basel II standards, the Operational Risk (OpRisk) advanced measurement approach is not prescriptive regarding the class of statistical model utilized to undertake capital estimation. It has however become well accepted to utilize a Loss Distributional Approach (LDA) paradigm to model the individual OpRisk loss processes corresponding to the Basel II Business line/event type. In this paper we derive a novel class of doubly stochastic α-stable family LDA models. These models provide the ability to capture the heavy tailed loss processes typical of OpRisk, whilst also providing analytic expressions for the compound processes annual loss density and distributions, as well as the aggregated compound processes' annual loss models. In particular we develop models of the annual loss processes in two scenarios. The first scenario considers the loss processes with a stochastic intensity parameter, resulting in inhomogeneous compound Poisson processes annually. The resulting arrival processes of losses under such a model will have independent counts over increments within the year. The second scenario considers discretization of the annual loss processes into monthly increments with dependent time increments as captured by a Binomial processes with a stochastic probability of success changing annually. Each of these models will be coupled under an LDA framework with heavy-tailed severity models comprised of α-stable severities for the loss amounts per loss event. In this paper we will derive analytic results for the annual loss distribution density and distribution under each of these models and study their properties.

1. Motivation

The modeling of Operational Risk (OpRisk) has taken a prominent place in financial quantitative measurement. This has occurred as a result of Basel II/Basel III regulatory requirements. As a result OpRisk has become increasingly important to the banking industry to address these regulatory standards in light of globalization, complex financial products and changes in information technology, combined with a growing number of high-profile operational loss events worldwide.

There was no widely accepted definition of OpRisk when the Basel Committee on Banking Supervision (BCBS) began discussions on OpRisk management at the end of the 1990s; see BCBS (1998). Often, OpRisk was defined as any risk not categorized as market or credit risk. Some banks defined it as the risk of loss arising from various types of human or technical error. Some earlier definitions can be found in a 1997 survey conducted by the British Bankers Association (BBA). In January 2001, the Basel Committee on Banking Supervision issued a proposal for a New Basel Capital Accord (referred to as Basel II) where OpRisk was formally defined as a new category of risk, in addition to market and credit risks, attracting a capital charge. In the working paper (BCBS, 2002) on the regulatory treatment of OpRisk and in the revised Basel II framework (BCBS, 2004), the following definition of OpRisk was adopted. "*Operational risk is defined as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk but excludes strategic and reputation risk*". This definition did not change in the latest version of Basel II framework, (BCBS, 2006, p. 144). The International Actuarial Association, IAA (2004), has adopted the same definition of operational risk in the capital requirements for insurance companies.

So OpRisk is indeed a broad category. The BCBS gives a further classification into seven event types of OpRisk (BCBS, 2006, Annex 9): Internal Fraud; External Fraud; Employment Practices and

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doi:10.1016/j.insmatheco.2011.08.007

Workplace Safety; Clients, Products and Business Practices; Damage to Physical Assets; Business Disruption and System Failure; Execution, Delivery and Process Management; which serves to further illustrate the disparate nature of events in this class. Reputation and strategic risk do not fall under the OpRisk umbrella, and market and credit risks are treated separately, but almost any other event that may result in a loss to a bank, including legal action, may be termed OpRisk.

Basel II considers three pillars, which, by their very nature, emphasize the importance of assessing, modeling and understanding OpRisk profiles. These three pillars are *minimum capital requirements* (refining and enhancing risk modeling frameworks), *supervisory review* of an institution's capital adequacy and internal assessment processes and *market discipline*, which deals with disclosure of information. Since this time, the discipline of OpRisk and its quantification have grown in prominence in the financial sector.

To illustrate just how significant OpRisk can be to a financial institution, one only needs to consider OpRisk related events such as the 1995 Barings Bank loss of around 1.3 billion GBP; the 2001 Enron loss of around 2.2 billion USD; the 2004 National Australia Bank loss of 360 m AUD; and the more recent Societe Generale loss of 4.9 billion Euros. Each of which demonstrates the severity of loss processes that should be modeled by OpRisk statistical models, providing strong motivation for heavy-tailed loss processes models such as those involving infinite mean and variance, as captured by the family of α-stable models considered in this paper.

The impact that such significant losses have had on the financial industry and its perceived stability combined with the Basel II regulatory requirements have significantly changed the view that financial institutions have regarding OpRisk. Under the three pillars of the Basel II agreement, set out in the framework, internationally active banks are required to set aside capital reserves against risk, to implement risk management frameworks and processes for their continual review and to adhere to certain disclosure requirements.

Whilst many OpRisk events occur frequently and with low impact (indeed, are 'expected losses'), others are rare, and their impact may be as extreme as the total collapse of the bank. The modeling and development of methodology to capture, classify and understand properties of operational losses is a new research area in the banking and finance sector.

There are three broad approaches that a bank may use to calculate its minimal capital reserve, as specified in the first pillar of the Basel II agreement. They are known as the Basic Indicator Approach, Alternative Standardized Approach and Advanced Measurement Approach (AMA). In this paper the approach considered is the AMA. The AMA is of interest since it is the most advanced framework with regards to statistical modeling.

A bank adopting the AMA must develop a comprehensive internal risk quantification system. This approach is the most flexible from a quantitative perspective, as banks may use a variety of methods and models, which they believe are most suitable for their operating environment and culture, provided they can convince the local regulator, (BCBS, 2006, p. 150–152). The key quantitative criteria is that a bank's models must sufficiently account for potentially high-impact rare events. In this paper we consider the idea of the loss distribution approach (LDA) which involves modeling the severity and frequency distributions over a predetermined time horizon, typically annual as specified in the APS115 section on soundness standards.

The fitting of frequency and severity distributions, as opposed to simply fitting a single parametric annual loss distribution, involves making the mathematical choice of working with compound distributions. This would seem to complicate the matter, since it is well known, that for most situations, analytical expressions for the distribution of a compound random variable are not attainable. However, the special classes of α-stable models developed in this paper overcome this complication.

Typically, the reason for modeling severity and frequency distributions separately then constructing a compound processes because some factors affect the frequency and others may affect the severity, see Willmot and Panjer (1985). Some of the key points relating to why this is important in most practical settings are that the expected number of operational losses will change as the company grows. Typically growth needs to be accounted for in forecasting the number of OpRisk losses in future years, based on previous years. This can easily be understood, when modeling is performed for frequency and severity separately. Economic inflationary effects can be directly factored into size of losses through scaling of the severity distribution. Insurance and the impacts of altering policy limits and excesses are easily understood by directly altering severity distributions. Changing recording thresholds for loss events and the impact this will have on the number of losses required to be recorded is transparent.

The most popular choices for frequency distributions are Poisson, binomial and negative binomial. The typical choices of severity distribution include exponential, Weibull, lognormal, generalized Pareto, the g-and-h family of distributions (Dutta and Perry, 2006; Peters and Sisson, 2006) and recently the α-stable family (Peters et al., 2011).

The most important processes to model accurately are those which have relatively infrequent losses. However, when these losses do occur they are distributed as a very heavy-tailed severity distribution. In particular we focus our analysis on the scenarios involving heavy-tailed severity models in the rare-event extreme consequence context, thereby providing analysis of the loss processes most likely to have significant consequences on a financial institution, those which may lead to ruin. This involves introducing to OpRisk modeling an important family of severity models, utilized in insurance claims reserving in Adler et al. (1998), given by the α-stable severity model. This family of severity model is flexible enough to incorporate light-tailed Gaussian loss models through to infinite mean, infinite variance severity loss models such as the Cauchy model.

There are many approaches which can be used to fit and incorporate expert opinion/scenario analysis for these parametric distributions and the approach adopted by a bank will depend on the data source being modeled and how much confidence one has in the data source. After the best-fitting models are selected, these are combined to produce compound processes for the annual loss distribution. From these compound processes, VaR and capital estimates may be derived.

Once compound processes have been fitted for each business unit and risk type, the next step is to aggregate these annual loss random variables for each individual {business line-event type} combination, and thus to obtain the institution-wide annual loss distribution. This paper will not address the issues associated with correlation and dependence modeling. For more information on typical approaches to introducing correlation in aggregation processes, including copula methods, correlation of frequency, severity or annual losses, see Cruz (2004) and Peters et al. (2009a). In the next section we present the details of the LDA modeling framework adopted in this paper.

2. Loss Distributional Approach model specifications

OpRisk LDA models are discussed widely in the literature; see e.g. Cruz (2004), Chavez-Demoulin et al. (2006), Frachot et al. (2004) and Shevchenko (2011). Under the LDA Basel II requirements, banks should quantify distributions for frequency and severity of OpRisk for each business line and event type over a one-year time horizon. In this section we begin by presenting