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Flexible dependence modeling of operational risk losses and its impact on total capital requirements

Eike Brechmann [a,1], Claudia Czado [a,2], Sandra Paterlini [b,*]

[a] Technische Universität München, Zentrum Mathematik, Parkring 13, 85748 Garching-Hochbrück, Germany

[b] European Business School, Dept. of Finance, Accounting and Real Estate, Gustav-Stresemann-Ring 3, 65189 Wiesbaden, Germany

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ABSTRACT

Operational risk data, when available, are usually scarce, heavy-tailed and possibly dependent. In this work, we introduce a model that captures such real-world characteristics and explicitly deals with heterogeneous pairwise and tail dependence of losses. By considering flexible families of copulas, we can easily move beyond modeling bivariate dependence among losses and estimate the total risk capital for the seven- and eight-dimensional distributions of event types and business lines. Using real-world data, we then evaluate the impact of realistic dependence modeling on estimating the total regulatory capital, which turns out to be up to 38% smaller than what the standard Basel approach would prescribe.

1. Introduction

The magnitude of operational losses observed in recent years and their potential systemic effects has pointed out the need of development of realistic and therefore often more sophisticated quantitative risk management models (see Basel Committee on Banking Supervision (2009b)). Among the main challenges in operational risk modeling, we have the presence of very heterogeneous losses, usually classified in a matrix of 56 risk classes (seven event types (ETs) × eight business lines (BLs)[3]; see Basel Committee on Banking Supervision (2006)), scarcity of data and large numbers of zero losses for some classes, short time series with extreme tails and the need to estimate quantiles at very high confidence levels. In fact, banks are required to calculate the minimum capital requirement as the 99.9%-Value-at-Risk of the loss distribution such that

$$\mathrm{MCR} = \mathrm{VaR}_{99.9\%}\left(\sum_{j=1}^{56} S_j\right), \tag{1.1}$$

where S_j is the aggregate loss of one of the 56 BL–ET combinations. It is clear that this quantity is influenced by the dependencies among the different risk classes. The standard approach of the Basel Committee on Banking Supervision (2006) recommends banks to marginally calculate the risk capital of all 56 BL–ET combinations and then determine the risk capital as the sum of these 56 figures, that is,

$$\mathrm{MCR}^{\mathrm{Basel}} = \sum_{j=1}^{56} \mathrm{VaR}_{99.9\%}(S_j). \tag{1.2}$$

This corresponds to the assumption of comonotonicity (perfect dependence) among all 56 BL–ET combinations, which is often perceived by banks as a worst-case scenario for the MCR, assuming that $\mathrm{MCR}^{\mathrm{Basel}} \geqslant \mathrm{MCR}$. However, due to the lack of subadditivity of the VaR measure for non-elliptical distributions (see Artzner et al. (1999)), it may also happen that $\mathrm{MCR} \geqslant \mathrm{MCR}^{\mathrm{Basel}}$. The question if

* Corresponding author. Tel.: +49 611 7102 1217.
E-mail addresses: eike.brechmann@tum.de (E. Brechmann), cczado@ma.tum.de (C. Czado), sandra.paterlini@ebs.edu (S. Paterlini).
[1] Tel.: +49 89 289 17425.
[2] Tel.: +49 89 289 17428.

[3] Business lines: 1. Corporate Finance, 2. Trading and Sales, 3. Retail Banking, 4. Commercial Banking, 5. Payment and Settlement, 6. Agency and Custody, 7. Asset Management, 8. Retail Brokerage. Event types: 1. Internal Fraud, 2. External Fraud, 3. Employment Practices & Workplace Safety, 4. Clients, Products & Business Practices, 5. Damage to Physical Assets, 6. Business Disruption & System Failures, 7. Execution, Delivery & Process Management.

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the standard Basel approach is appropriate has therefore been raised many times by practitioners and researchers.

Regulators allow then, with due diligence, explicit dependence modeling under the Advanced Measurement Approach (AMA).[4] The supervisory guidelines for the AMA banks (Basel Committee on Banking Supervision, 2011) underline that dependence modeling for operational risk is an evolving area, where many approaches are currently used by banks with potential impact on the capital requirements. Results from the 2008 Loss Data Collection Exercises (Basel Committee on Banking Supervision, 2009a) suggest that, among the AMA banks only 17% use correlation coefficients, while most rely on copulas (43%), with a preference for Gaussian copulas, and 31% AMA banks use other methods.

Research is high then on the regulators agenda to avoid spurious differences in exposure estimates (see Basel Committee on Banking Supervision (2011, Paragraph 224)) and to provide sound guidelines for dependence modeling in operational risk, explicitly suggesting to move beyond Gaussian copulas and correlations coefficients. Theoretical and empirical evidence so far has mostly supported the idea that the assumption of perfect positive dependence is unduly strong and using internally determined correlations, as Basel II allows, could lead to lower the risk capital requirements while still providing adequate coverage for future losses (see Artzner et al. (1999), Chapelle et al. (2008) and Frachot et al. (2004)). However, recently Mittnik et al. (2013) have also shown that, despite only for a small number of risk classes, modeling bivariate dependence could also lead to increase the risk capital. Especially tail dependence, which cannot be captured by Pearson's correlation coefficient, should not be ignored. What is the total impact on risk capital of explicit dependence modeling among all BLs and ETs is still, to our knowledge, a question with no answer, as estimating realistic multivariate operational risk management models with more than two cells can be computationally challenging and data are often scarce for model validation.

In this work, we aim at analyzing how much the risk capital estimate may change when modeling dependencies in multivariate settings. That is, we consider the total impact of explicit dependence modeling within the eight- or seven-dimensional BL and ET distributions by introducing a statistical model, which allows to explicitly consider the presence of extreme tails, heterogeneous pairwise dependence and large numbers of zero observations. In particular, we propose a flexible approach that, inspired by the work of Deb et al. (2009) on drug expenditures and Erhardt and Czado (2012) on dependent health insurance claims, directly models the dependence between the aggregate losses in BL–ET combinations using copulas. Since the non-occurrence of losses (zero events) also conveys information about the dependence characteristics, we explicitly incorporated it to allow a more accurate assessment of dependence. Finally, given that no excessive data aggregation is required, parameter estimation can be based on the maximum amount of available observations.

By using real-world data from the Italian Database of Operational Losses (DIPO) in the period from January 2003 to June 2011, we can provide new and much needed insights on the impact of different dependence modeling strategies on total capital requirements and their validation on real-world data. In fact, our results suggest that explicitly modeling dependence can lead to a reduction, as often expected, of the total regulatory capital, which might turn out to be up to 38% smaller than what the Basel comonotonicity approach would prescribe.

The paper is organized as follows. Section 2 describes the general modeling framework by discussing the key components of our modeling strategy. Dependence modeling of positive losses is treated in Section 3, after introducing the key properties we consider as relevant to compare the four major copula classes that we investigate for multivariate operational risk modeling. Marginal modeling and a detailed description of the considered copula families are reported in Appendices A and B, respectively. Modeling dependence among zero events as an additional model component is described in Section 4, and the computation of risk measures is subsequently discussed in Section 5. Section 6 finally provides the results of the empirical investigation on real-world data, while Section 7 concludes.

2. Zero-inflated dependence model

Common characteristics of operational risk data can be summarized as follows. First, if losses are modeled on a weekly or monthly basis, it may frequently occur that there are no losses observed for a particular BL or ET. An excessive number of zero losses is called *zero inflation*. Second, distributions of operational losses per BL or ET may be *heavy-tailed*, that is, there is a significant probability of extreme losses that has to be taken into account. Third, different BLs and ETs are not independent. Most importantly, the type of dependence, especially the so-called *tail dependence*, may have a large impact on risk capital estimates.

While the heavy tails in marginal distributions have already been extensively discussed in the literature (see for example Chavez-Demoulin et al. (2006) and Gourier et al. (2009)), appropriate zero-inflated dependence models for aggregate operational losses have, to our knowledge, not yet been proposed.

Copulas have been established as the fundamental concept of statistical dependence modeling. A d-dimensional copula is a multivariate distribution function on $[0,1]^d$ with uniform marginal distribution functions. Its central role in dependence modeling is due to the theorem by Sklar (1959), which states that any multivariate distribution can be decomposed into its margins and a copula. More precisely, let $\boldsymbol{X} = (X_1,\ldots,X_d)' \sim F_{1,\ldots,d}$ with marginal distributions $F_1,\ldots,F_d$, then

$$F_{1,\ldots,d}(x_1,\ldots,x_d) = C_{1,\ldots,d}(F_1(x_1),\ldots,F_d(x_d)), \quad x_1,\ldots,x_d \in \mathbb{R}\cup\{-\infty,\infty\}, \tag{2.1}$$

where $C_{1,\ldots,d}$ is a d-dimensional copula. If $\boldsymbol{X}$ is a continuous random vector, then the copula $C_{1,\ldots,d}$ is unique and the multivariate density $f_{1,\ldots,d}$ of $\boldsymbol{X}$ can be decomposed as

$$f_{1,\ldots,d}(x_1,\ldots,x_d) = c_{1,\ldots,d}(F_1(x_1),\ldots,F_d(x_d))f_1(x_1),\ldots,f_d(x_d), \tag{2.2}$$

where $c_{1,\ldots,d}$ is the copula density and $f_1,\ldots,f_d$ are the marginal densities of $f_{1,\ldots,d}$. Comprehensive reference books on copulas are Joe (1997) and Nelsen (2006).

Now, suppose that we want to model the multivariate distribution of d BLs, ETs or cells of the 7×8 BL–ET matrix. For brevity, we henceforth always speak of d cells with $d\in\{7,8,56\}$. Let $S_j \geqslant 0,\ j=1,\ldots,d$, denote the aggregate loss of the jth cell. Then, we explicitly model the presence of zero inflation in the aggregate loss by defining the following binary random variable $W_j \sim P_{W_j}$ for each cell $j\in\{1,\ldots,d\}$ as

$$W_j := \begin{cases} 1, & \text{zero loss in cell } j \\ 0, & \text{otherwise} \end{cases}.$$

That is, W_j is the zero inflation component of the otherwise positive continuous part of S_j, which we denote by $S_j^+ > 0$. We obtain the following decomposition:

[4] Paragraph 669d in Basel Committee on Banking Supervision (2006): *"Risk measures for different operational risk estimates must be added for purposes of calculating the regulatory minimum capital requirement. However, the bank may be permitted to use internally determined correlations in operational risk losses across individual operational risk estimates, provided it can demonstrate to the satisfaction of the national supervisor that its systems for determining correlations are sound, implemented with integrity, and take into account the uncertainty surrounding any such correlation estimates (particularly in periods of stress). The bank must validate its correlation assumptions using appropriate quantitative and qualitative techniques."*