Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 321 (2003) 286–299
PHYSICA A
www.elsevier.com/locate/physa

From Brownian motion to operational risk: Statistical physics and financial markets

Johannes Voit*,[1]

Theoretische Physik 1, Universität Bayreuth, D-95440 Bayreuth, Germany

Abstract

High-frequency returns of the DAX German blue chip stock index are used to test geometric Brownian motion, the standard model for financial time series. Even on a 15-s time scale, the linear correlations of DAX returns have a zero-time delta function which carries 90% of the weight, while the remaining 10% are positively correlated with a decay time of 53 s and negatively correlated on a 9.4-min scale. The probability density of the returns possesses fat tails with power laws whose exponents continuously increase with time scales. It is suggested that hydrodynamic turbulence may provide a phenomenological framework for the description of these data, and at the same time, open a way to use them for risk-management purposes, e.g. option pricing and hedging. Option pricing also is the cornerstone of credit valuation, an area of much practical importance not considered explicitly in most other physics-inspired papers on finance. Finally, operational risk is introduced as a new risk category currently emphasized by regulators, which will become important in many banks in the near future.

Keywords: Brownian motion; Turbulence; Financial markets; DAX stock index; Option pricing; Credit default risk; Basel II capital accord; Operational risk

1. Introduction

Interactions between physics and finance have a long history. Apart from the investment ventures of physicists into stock markets, such as Isaac Newton's losing a fortune in the South Sea bubble in summer 1720, the one-dimensional random walk

* Tel.: +49-921-55-31-85; fax: +49-921-55-29-91.
E-mail address: Johannes.Voit@dsgv.de (J. Voit).
URL: http://www.phy.uni-bayreuth.de/~btp314

[1] Present address: Deutscher Sparkassen- und Giroverband, Abteilung C IV Controlling, Simrockstraße 4, D-53113 Bonn, Germany.

0378-4371/03/$ - see front matter
doi:10.1016/S0378-4371(02)01783-1

was described successfully almost simultaneously in finance by Bachelier in 1900 [1,2] and in physics by Einstein in 1905 [3]. Bachelier's work in finance was forgotten and rediscovered in the 1950s, among others, by physicists [2,4]. Later, Mandelbrot applied Lévy-stable stochastics processes and multifractal processes to the description of financial time series [5,6].

The application of statistical physics to the description of financial markets [7–10] and economics [11–14] has become more systematic, broader and deeper during the last couple of years. Topics discussed include:

- statistical properties of financial and economic time series, such as
  - probability densities and scaling of price changes,
  - linear and nonlinear correlations in and among time series,
  - universality in financial time series, and their stochastic modeling;
- applications to option pricing and hedging;
- applications to portfolio theory;
- simulation of microscopic market models;
- theory and prediction of stock market crashes.

This new direction of research is driven by increasing evidence that an improved understanding of financial markets can be gained by using parallels to phenomena in nature such as normal and anomalous diffusion, earthquakes, phase transitions, turbulence, highly excited nuclei, etc., and the use of the specific models and techniques that have been devised for their description [7].

In this paper, we will *not* review this work. Comprehensive reviews can be found in a number of books which have appeared over the last years [7–10]. Instead, we will sketch some of the important questions, both for fundamental and applied research, which have driven past research or which are likely to become important in the future. In some cases, we present new material from our own research. In other more forward-looking cases, data are not yet available, and we limit ourselves to the outline of directions of research.

2. Geometric Brownian motion—the standard model of asset returns

Fig. 1 displays the history, on a 15-s time scale, of the DAX German blue chip stock index for the years 1999 and 2000. The DAX index is composed of the 30 biggest companies in Germany in terms of market capitalization, and fixed every 15 s.

Speculative investors will ask whether profitable investments can be made both during the upward and during the downward moves of the market, and if such investments can be leveraged, i.e., the returns of an investment are bigger than those of the market, what would be the success rates of such investments. These questions are not new, and basically are those at the origin of the Ph.D. thesis of the French mathematician Louis Bachelier [1]. For risk-management purposes, one would rather ask if there are strategies to protect oneself from losses arising from adverse market moves once an open position cannot be closed for strategic or commercial reasons (hedging).